Vector Analysis Theorems

1. The **Fundamental Theorem of Calculus for Curves** states that if \( \vec{r}(t) \) is a nice curve in \( \mathbb{R}^n \) and \( f \) is a nice function in \( \mathbb{R}^n \) then

\[
\int_{A}^{B} \nabla f \cdot \, d\vec{s} = f(B) - f(A)
\]

2. **Green’s Theorem** states that if \( R \) is a nice region in \( \mathbb{R}^2 \) and \( \partial R \) is its boundary curve traversed counterclockwise and \( P \) and \( Q \) are nice functions on \( R \) then

\[
\iint_{R} \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dx \, dy = \oint_{\partial R} P \, dx + Q \, dy
\]

a. **2D Stokes’ Theorem** states that if \( R \) is a nice region in \( \mathbb{R}^2 \) and \( \partial R \) is its boundary curve traversed counterclockwise and \( \vec{F} = (P(x,y), Q(x,y), 0) \) is a nice vector field on \( R \) then

\[
\iint_{R} \vec{\nabla} \times \vec{F} \cdot \hat{k} \, dx \, dy = \oint_{\partial R} \vec{F} \cdot d\vec{s}
\]

b. **2D Gauss’ Theorem** states that if \( R \) is a nice region in \( \mathbb{R}^2 \) and \( \partial R \) is its boundary curve traversed counterclockwise and \( \vec{G} = (Q(x,y), -P(x,y), 0) \) is a nice vector field on \( R \) then

\[
\iint_{R} \vec{\nabla} \cdot \vec{G} \, dx \, dy = \oint_{\partial R} \vec{G} \cdot d\vec{n}
\]

3. **Stokes’ Theorem** states that if \( S \) is a nice surface in \( \mathbb{R}^3 \) and \( \partial S \) is its boundary curve traversed counterclockwise as seen from the tip of the normal to \( S \) and \( \vec{F} \) is a nice vector field on \( S \) then

\[
\iint_{S} \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{s}
\]

4. **Gauss’ Theorem** states that if \( V \) is a volume in \( \mathbb{R}^3 \) and \( \partial V \) is its boundary surface oriented outward from \( V \) and \( \vec{F} \) is a nice vector field on \( V \) then

\[
\iiint_{V} \vec{\nabla} \cdot \vec{F} \, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}
\]