Multiple Choice & Work Out: (5 points each)

1. Find the equation of the line through the point \( P = (4, 3, 2) \) in the direction \( \vec{v} = (2, 1, -1) \). Where does this line pass through the \( xy \)-plane?
   a. \((8, 5, 0)\) Correct Choice
   b. \((1, 2, 0)\)
   c. \((8, -5, 0)\)
   d. \((1, -2, 0)\)
   e. \(\left(4, \frac{5}{2}, 0\right)\)

   The line is: \( X = P + t\vec{v} \) or \((x, y, z) = (4 + 2t, 3 + t, 2 - t)\).
   This intersects the \( xy \)-plane when \( z = 0 \) or \( 2 - t = 0 \) or \( t = 2 \).
   So \( x = 4 + 2t = 8 \) and \( y = 3 + t = 5 \).

2. Find the equation of the plane through the point \( P = (1, 3, 3) \) with normal \( \vec{N} = (4, 2, -2) \). Where does this plane pass through the \( z \)-axis?
   a. \((0, 0, -2)\) Correct Choice
   b. \((0, 0, -1)\)
   c. \((0, 0, 1)\)
   d. \((0, 0, 2)\)
   e. \((0, 0, 4)\)

   The line is: \( \vec{N} \cdot X = \vec{N} \cdot P \) or \( 4x + 2y - 2z = 4 \cdot 1 + 2 \cdot 3 - 2 \cdot 3 = 4 \).
   This intersects the \( z \)-axis when \( x = y = 0 \). So \( -2z = 4 \) or \( z = -2 \).

3. Classify the curve \( x^2 - y^2 - 6x - 4y = -4 \)
   a. circle with center \((3, -2)\)
   b. circle with center \((-3, 2)\)
   c. hyperbola opening left and right Correct Choice
   d. hyperbola opening up and down
   e. parabola with vertex \((-3, 2)\)

   Complete squares: \( (x^2 - 6x + 9) - (y^2 + 4y + 4) = -4 + 9 - 4 = 1 \)
   or \( (x - 3)^2 - (y + 2)^2 = 1 \) which is a hyperbola.
   Since \( (x - 3)^2 = 1 + (y + 2)^2 \) we have \( (x - 3)^2 \geq 1 \).
   So the hyperbola opens left and right.
4. Classify the surface \( x^2 + y^2 - 4x - 4y - z = -4 \)
   
   a. elliptic paraboloid opening up Correct Choice
   b. elliptic paraboloid opening down
   c. hyperboloid of 1 sheet
   d. hyperboloid of 2 sheets
   e. hyperbolic paraboloid

Complete squares: \( (x^2 - 4x + 4) + (y^2 + 4y + 4) - z = -4 + 4 + 4 = 4 \)
   or \( z = (x - 2)^2 + (y - 2)^2 - 4 \) which is a paraboloid.
Since the coefficients of \( x^2 \) and \( y^2 \) are both positive, the paraboloid is elliptic, opening up.

5. Find the point where the line \( \frac{x - 1}{-1} = \frac{y - 5}{2} = z - 7 \) intersects the plane \( 3x - 2y + z = 12 \).
   HINT: Use the line to write \( x \) and \( y \) as functions of \( z \).
Solve this on the back of the Scantron. Show all work.

\[
x - 1 = -1(z - 7) \quad x = -z + 8 \quad y - 5 = 2(z - 7) \quad y = 2z - 9
\]

Plug into the plane:
\[
3(-z + 8) - 2(2z - 9) + z = 11 \quad \text{or} \quad -3z + 24 - 4z + 18 + z = 12 \quad \text{or} \quad -6z = -30
\]
So \( z = 5 \quad x = -z + 8 = -5 + 8 = 3 \quad y = 2z - 9 = 2 \cdot 5 - 9 = 1 \)
So the point is: \( (x, y, z) = (3, 1, 5) \)

Check: \( \frac{x - 1}{-1} = \frac{3 - 1}{-1} = -2 \quad \frac{y - 5}{2} = \frac{1 - 5}{2} = -2 \quad z - 7 = 5 - 6 = -2 \) Good
\( 3x - 2y + z = 3 \cdot 3 - 2 \cdot 1 + 5 = 12 \) Good