Multiple Choice & Work Out: (5 points each)

1. Find the equation of the plane tangent to the surface $ze^{xy^2} = 3$ at the point $(2, 1, 3)$. Its $z$-intercept is:
   
   a. 3
   b. -3
   c. 15 Correct Choice
   d. -15
   e. 0

   $P = (2, 1, 3) \quad F = ze^{xy^2} \quad \vec{v}F = \langle yze^{xy^2}, xze^{xy^2}, e^{xy^2} \rangle \quad \vec{N} = \vec{v}F \bigg|_P = \langle 3, 6, 1 \rangle$

   Tangent plane is $\vec{N} \cdot X = \vec{N} \cdot P$ or $3x + 6y + z = 3 \cdot 2 + 6 \cdot 1 + 1 \cdot 3 = 15$
   or $z = 15 - 3x - 6y$ The $z$-intercept is 15.

2. Find the equation of the line perpendicular to the surface $ze^{xy^2} = 3$ at the point $(2, 1, 3)$. It intersects the $xy$-plane at:
   
   a. $(7, 17, 0)$
   b. $(-7, -17, 0)$ Correct Choice
   c. $(11, 19, 0)$
   d. $(-11, -19, 0)$
   e. $(11, 19, 6)$

   $P = (2, 1, 3) \quad F = ze^{xy^2} \quad \vec{v}F = \langle yze^{xy^2}, xze^{xy^2}, e^{xy^2} \rangle \quad \vec{v} = \vec{v}F \bigg|_P = \langle 3, 6, 1 \rangle$

   Normal line is $X = P + t\vec{v} = (2, 1, 3) + t(3, 6, 1)$ or $(x, y, z) = (2 + 3t, 1 + 6t, 3 + t)$
   The line intersects the $xy$-plane when $z = 0$ or $3 + t = 0$ or $t = -3$
   $(x, y, z) = (2 + 3(-3), 1 + 6(-3), 3 + (-3)) = (-7, -17, 0)$. 
3. If the temperature in a room is given by \( T = 75 + xy^2z \) and a fly is located at \((2,1,3)\), in what unit vector direction should the fly fly in order to decrease the temperature as fast as possible?

   a. \( \langle 3, 12, 2 \rangle \)
   b. \( \langle 3, -12, 2 \rangle \)
   c. \( \langle -3, -12, -2 \rangle \)
   d. \( \frac{1}{\sqrt{157}} \langle 3, 12, 2 \rangle \)
   e. \( \frac{1}{\sqrt{157}} \langle -3, -12, -2 \rangle \) Correct Choice

\( \vec{V}T = \langle y^2z, 2xyz, xy^2 \rangle \quad \vec{v} = \left. \vec{V}T \right|_{(2,1,3)} = \langle 3, 12, 2 \rangle \quad |\vec{v}| = \sqrt{9 + 144 + 4} = \sqrt{157} \)

Direction of Max increase is \( \hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{\sqrt{157}} \langle 3, 12, 2 \rangle \).

Direction of Max decrease is \( -\hat{v} = -\frac{1}{\sqrt{157}} \langle 3, 12, 2 \rangle \).

4. Which of the following is NOT a critical point of \( f(x,y) = (2x - x^2)(4y - y^2) \)?

   a. \((0,0)\)
   b. \((0,4)\)
   c. \((1,2)\)
   d. \((2,0)\)
   e. \((-2,4)\) Correct Choice

\( f_x = (2 - 2x)(4y - y^2) = 0 \quad f_y = (2x - x^2)(4 - 2y) = 0 \)

From \( f_x = 0 \), either \( x = 1 \) or \( y = 0 \) or \( y = 4 \)

Case \( x = 1 \): From \( f_y = 0 \), \( (4 - 2y) = 0 \) \( \Rightarrow \) \( y = 2 \)

Case \( y = 0 \): From \( f_y = 0 \), \( (2x - x^2)4 = 0 \) \( \Rightarrow \) \( x = 0 \) or \( x = 2 \)

Case \( y = 4 \): From \( f_y = 0 \), \( (2x - x^2)(-4) = 0 \) \( \Rightarrow \) \( x = 0 \) or \( x = 2 \)

The critical points are: \((1,2)\), \((0,0)\), \((2,0)\), \((0,4)\), \((2,4)\)

OR Simply plug each answer into \( f_x \) and \( f_y \)

5. Find 3 numbers \( a \), \( b \) and \( c \) whose sum is 80 for which \( ab + 2bc + 3ac \) is a maximum.

Solve on the back of the Scantron.

We need to maximize \( f = ab + 2bc + 3ac \) subject to the constraint \( a + b + c = 80 \).

\( c = 80 - a - b \) \quad \( f = ab + 2b(80 - a - b) + 3a(80 - a - b) = 240a + 160b - 3a^2 - 2b^2 - 4ab \)

\( f_a = 240 - 6a - 4b = 0 \) \quad \( f_b = 160 - 4b - 4a = 0 \)

\( 6a + 4b = 240 \) \quad \( 4a + 4b = 160 \)

Subtract: \( 2a = 80 \) \( a = 40 \)

Substitute back: \( 4b = 160 - 4a = 0 \) \( b = 0 \)

\( c = 80 - a - b = 40 \)

So \( a = 40, \ b = 0, \ c = 40 \)