Multiple Choice: (7 points each)

1. Consider the line through the point \( P = (4, 4, 4) \) which is perpendicular to the plane \( x + 2y + 3z = 7 \). Its tangent vector is

   a. \((3, 2, 1)\)
   b. \((1, 2, 3)\)
   c. \((7, 6, 5)\)
   d. \((5, 6, 7)\)
   e. \((4, 4, 4)\)

2. Find the plane tangent to the hyperbolic paraboloid \( x - yz = 0 \) at the point \( P = (6, 3, 2) \). Which of the following points does not lie on this plane?

   a. \((-6, 0, 0)\)
   b. \((0, 3, 0)\)
   c. \((0, 0, 2)\)
   d. \((1, -1, -1)\)
   e. \((-1, 1, 1)\)

3. Duke Skywater is flying the Millenium Eagle through a polaron field. His galactic coordinates are \((2300, 4200, 1600)\) measured in lightseconds and his velocity is \( \vec{v} = (2, 3, 4) \) measured in lightseconds per second. He measures the strength of the polaron field is \( p = 274 \) milliwookies and its gradient is \( \nabla p = (3, 2, 2) \) milliwookies per lightsecond. Assuming a linear approximation for the polaron field and that his velocity is constant, how many seconds will Duke need to wait until the polaron field has grown to \( 286 \) milliwookies?

   a. 2
   b. 3
   c. 4
   d. 6
   e. 12
4. Consider the surface $S$ parametrized by $\vec{R}(u, v) = (u + v, u - v, uv)$ for $0 \leq u \leq 2$ and $0 \leq v \leq 4$. Compute $\int_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = (y, x, y)$.

   a. $-32$
   b. $-16$
   c. $16$
   d. $32$
   e. $64$

5. Consider the surface $S$ parametrized by $\vec{R}(u, v) = (u + v, u - v, uv)$. Find the plane tangent to this surface at the point $P = \vec{R}(1, 2) = (3, -1, 2)$. Which of the following points does not lie on this plane?

   a. $(3, 0, 0)$
   b. $(0, 4, 0)$
   c. $(0, 0, -2)$
   d. $(1, 1, 0)$
   e. $(0, 6, 1)$

6. Compute $\int_S (-x^2y^2 \, dx + 2xy^3 \, dy)$ over the complete boundary of the semicircular area $0 \leq y \leq \sqrt{4 - x^2}$ traversed counterclockwise.

   a. $0$
   b. $16$
   c. $\frac{4}{5}$
   d. $\frac{80}{5}$
   e. $\frac{128}{5}$

7. Compute $\int_S \left( \frac{x^3z^2}{3} \, dy \, dz + \frac{y^3z^2}{3} \, dz \, dx + \frac{z^5}{5} \, dx \, dy \right)$ over the complete surface of the sphere $x^2 + y^2 + z^2 = 4$ with outward normal.

   a. $\frac{512\pi}{21}$
   b. $\frac{32\pi^2}{5}$
   c. $\frac{128\pi}{5}$
   d. $\frac{16\pi}{3}$
   e. $\frac{256\pi}{15}$
8. (15 points) Find the point in the first octant on the surface \( z = \frac{32}{x^4y^2} \) which is closest to the origin.
9. (10 points) Compute \( \int \int_{R} x \, dA \)

over the region \( R \) in the first quadrant bounded by the curves

\[ y = x^2, \quad y = x^4 \quad \text{and} \quad y = 16. \]
10. (15 points) Find the mass and center of mass of the solid below the paraboloid \( z = 4 - x^2 - y^2 \) above the \( xy \)-plane, if the density is \( \delta = x^2 + y^2 \). (11 points for setting up the integrals and the final formula.)
11. (15 points) Find the area and centroid of the \textbf{right} leaf of the rose \\
\[ r = 2\cos^2\theta. \]
(12 points for setting up the integrals and the final formula.)