1. (10 points) Let $P^0_2$ be the subset of $P_2$ consisting of those polynomials of degree 2 or less whose constant term is zero. In particular

$$P^0_2 = \{ p = ax + bx^2 \}$$

a. (8) Show $P^0_2$ is a subspace of $P_2$.

b. (2) What is the dimension of $P^0_2$? Why?
2. (20 points) Again let \( P^0_2 \) be the subset of \( P_2 \) consisting of those polynomials of degree 2 or less whose constant term is zero. Consider the function \( \langle *, * \rangle \) of two polynomials given by
\[
\langle p, q \rangle = \int_0^1 \frac{4pq}{x^2} \, dx.
\]

a. (5) Show the function \( \langle *, * \rangle \) is an inner product on \( P^0_2 \).

b. (10) Apply the Gram Schmidt procedure to the basis \( p_1 = x, \ p_2 = x^2 \) to produce an orthogonal basis \( q_1, q_2 \) and an orthonormal basis \( r_1, r_2 \).

\[
q_1 = \quad q_2 = \quad r_1 = \quad r_2 =
\]

\( \Leftarrow \quad \Leftarrow \quad \Leftarrow \quad \text{USE THE BACK OF THE OPPOSITE PAGE.} \)

c. (5) Find the change of basis matrices \( C_{p \rightarrow q} \) and \( C_{p \rightarrow r} \).
3. (20 points) Again let $P^0_2$ be the subset of $P_2$ consisting of those polynomials of degree 2 or less whose constant term is zero. Consider the function $L : P^0_2 \to P_2$ given by

$$L(p) = p - \frac{dp}{dx}.$$ 

a. (4) Show the function $L$ is linear.

b. (6) Find the kernel of $L$. Give a basis.

c. (6) Find the image of $L$. Give a basis.

d. (2) Is $L$ onto? Why?

e. (2) Is $L$ one-to-one? Why?
4. (20 points) Again let $P_2^{0}$ be the subset of $P_2$ consisting of those polynomials of degree 2 or less whose constant term is zero. Again consider the function $L : P_2^{0} \rightarrow P_2$ given by 
\[ L(p) = p - \frac{dp}{dx}. \]

a. (10) Find the matrix of $L$ relative to the bases 
\[ p_1 = x, \quad p_2 = x^2 \quad \text{for} \quad P_2^{0} \quad \text{and} \quad e_1 = 1, \quad e_2 = x, \quad e_3 = x^2 \quad \text{for} \quad P_2. \]

Call it $A$.

b. (5) Find the matrix of $L$ relative to the bases 
\[ r_1, r_2 \quad \text{for} \quad P_2^{0} \quad \text{and} \quad e_1 = 1, \quad e_2 = x, \quad e_3 = x^2 \quad \text{for} \quad P_2 \]

where $r_1, r_2$ is the orthonormal basis you found in problem 2. Call it $B$.

c. (5) Recompute $B$ by another method.
5. (30 points) Do this problem, if you did the Volume of Desserts or Planet X Project.

Find the \( z \)-component of the center of mass of the apple whose surface is given in spherical coordinates by
\[
\rho = 1 - \cos \varphi
\]
and whose density is 1.

HINT: The \( \varphi \)-integrals can be done using the substitution
\[
u = 1 - \cos \varphi.
\]
6. (30 points) Do this problem, if you did the Interpretation of Div and Curl Project.

Find the divergence of the vector field \( \vec{F} = (xz^2, yz^2, 0) \) at the point \((x, y, z) = (0, 0, c)\).

a. by using the derivative definition:

b. by using the integral definition:

HINTS: For a sphere of radius \( \rho \) centered at \((a, b, c)\), if you use standard spherical coordinates, the normal vector is

\[
\vec{N} = \left( \rho^2 \sin^2 \varphi \cos \theta, \rho^2 \sin^2 \varphi \sin \theta, \rho^2 \cos \varphi \sin \varphi \right)
\]

The \( \varphi \)-integral can be done using the substitution \( u = \cos \varphi \).

You can ignore terms in the integral proportional to \( \rho^n \) with \( n > 3 \) since they drop out of the limit.
7. (30 points) Do this problem, if you did the Gauss’ and Ampere’s Laws Project.

Find the total charge in the cylinder \( x^2 + y^2 \leq a^2, \quad 0 \leq z \leq 1 \) if the electric field is

\[
\vec{E} = \frac{\vec{r}}{r^2} = \frac{\vec{r}}{r^2} = \left( \frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2}, 0 \right)
\]

where \( \vec{r} = (x, y, 0) \) and \( r = \sqrt{x^2 + y^2} \).

a. using the derivative form of Gauss’ Law.

b. using the integral form of Gauss’ Law.

c. What do these results tell you about the location of the electric charge? Why?