1. (15 points) Compute

\[
\text{det} \begin{pmatrix}
2 & 5 & 4 & -1 & 4 \\
0 & 1 & -2 & 1 & 2 \\
1 & 3 & 0 & -2 & 1 \\
2 & 6 & 3 & -4 & -1 \\
-2 & -6 & -3 & 0 & 1
\end{pmatrix}
\]
2. (15 points) Use row operations on the augmented matrix to solve the system of equations

\[ \begin{align*}
2u + 4v - 2w - 2x + 2y - 4z &= 4 \\
u + 2v - 2w - 3x + y - 3z &= -3 \\
3u + 6v - 2w - x + 3y - 5z &= b
\end{align*} \]

a. For what value(s) of \( b \) do there exist solutions.

\[ b = \]

b. For those value(s) of \( b \) write the set of all solutions in parametric form.

\[ u = x = \]

\[ v = y = \]

\[ w = z = \]

c. Interpret the solution set as a \( k \)-plane in \( \mathbb{R}^n \) for some \( k \) and \( n \).
3. (15 points) Find the equation of the plane tangent to the graph of the function 
\[ f(x,y) = 3x \sin y - 2y \cos x \] at the point \( (x,y) = \left(0, \frac{\pi}{2}\right) \).

4. (15 points) Find the equation of the line perpendicular to the surface 
\[ F(x,y,z) = x^2 y + y^3 z + z^3 x = 29 \] at the point \( P = (x,y,z) = (3,2,1) \).
5. (20 points) Consider the matrix \( A = \begin{pmatrix} -2 & -5 & 1 \\ 1 & 3 & -1 \\ 3 & 7 & -1 \end{pmatrix} \).

a. Find \( A^{-1} \) or show it does not exist.

b. Consider the equations \( AX = 0 \), i.e.

\[ -2x - 5y + z = 0 \]
\[ x + 3y - z = 0 \]
\[ 3x + 7y - z = 0 \]

How many solutions are there?
Circle one: (Do not solve the equations.)

No Solutions    Unique Solution    \( \infty \)-Many Solutions

Explain why:
6. (20 points) Consider the matrix \( A = \begin{pmatrix} -2 & -5 & 1 \\ 1 & 3 & -1 \\ 3 & 7 & 0 \end{pmatrix} \).

a. Find \( A^{-1} \) or show it does not exist.

b. Consider the equations \( AX = \mathbf{j} \), i.e.

\[
-2x - 5y + z = 0 \\
x + 3y - z = 1 \\
3x + 7y = 0
\]

How many solutions are there?
Circle one:

- No Solutions 
- Unique Solution 
- \( \infty \)-Many Solutions

Find all solutions if there are any.