We consider the system of linear equations $Ax = b$, where $A$ is a nonsingular matrix.

1. We consider the case when $A$ is Hermitian and positive definite matrix in $\mathbb{C}^{n \times n}$ and $x^0$ is an arbitrary vector in $\mathbb{C}^n$. As usual we split $A = D - E - F$, where $D$ is the diagonal of $A$, $E$ is the strictly lower triangular matrix and $F$ is strictly upper triangular matrix.

   Consider the following two step iteration consisting of forward and backward Gauss-Seidel sweeps (called symmetric Gauss-Seidel):

   $$(D - F)x^{m+1} = Ex^m + b, \quad (D - E)x^{m+2} = Fx^{m+1} + b \text{ for } m = 0, 2, \ldots.$$  

   (a) Present this as a product iteration in the form $x^{s+1} = Gx^s + Bb$;  

   (b) Prove that the iteration converges.

2. Following the above idea formulate a symmetric SOR iteration (called SSOR).

3. Let $A$ be strongly diagonally dominant, i.e. $|a_{ii}| > \sum_{j \neq i} |a_{ij}|$ for $j = 1, \ldots, n$. Prove that:

   (a) Jacobi iteration converges;  

   (b) Gaus-Seidel iteration converges.