13.4: Polar Coordinates

The polar coordinate system consists of:

- the pole (or origin) labeled $O$;
- the polar axis which is a ray starting at $O$ (usually drawn horizontally to the right);

The polar coordinates $(r, \theta)$ of a point $P$:

- $\theta$ is the angle between the polar axis and the line $OP$ (the angle is positive if measured in counterclockwise direction from the polar axis);
- $r$ is the distance from $O$ to $P$.

EXAMPLE 1. Plot the points whose polar coordinates are given:

(a) $(1, \pi/3)$  
(b) $(5, -\pi/2)$.

The connection between polar and Cartesian coordinates:

\[
\cos \theta = \\
\sin \theta = \\
x = \\
y = \\
r^2 = \\
\tan \theta =
\]
REMARK 2. In converting from the Cartesian to polar coordinates we must choose \( \theta \) so that the point \((r, \theta)\) lies in the correct quadrant.

EXAMPLE 3. Convert the point \((4, \pi/6)\) from polar to Cartesian coordinates.

EXAMPLE 4. Represent the point with Cartesian coordinates \((-10, 10)\) in terms of polar coordinates.

EXAMPLE 5. Find the distance between the points \(A(2, \pi/6)\) and \(B(3, \pi/3)\) in polar coordinates.
EXAMPLE 6. What curve is represented by the polar equation $r = 12$?

EXAMPLE 7. What curve is represented by the polar equation $\theta = \pi/4$?

EXAMPLE 8. Sketch the region in the Cartesian plane consisting of points whose polar coordinates satisfy the following conditions: $1 \leq r \leq 2$, $0 \leq \theta \leq \pi$. 
EXAMPLE 9. Sketch the curve with polar equation $r = 2 \sin \theta$.

EXAMPLE 10. Sketch the curve with polar equation $r = 2 \cos \theta$. 
EXAMPLE 11. Sketch the curve $r = 1 + \cos \theta$.

EXAMPLE 12. Sketch the curve $r^2 = 4 \cos 2\theta$. 