7.4: Work

PROBLEM: Find the amount of work that is done by a force in moving an object.

Solution:
- Case 1: constant force.
  Work done in moving an object a distance \( d \) meters is given by
  \[
  W = Fd.
  \]

  In the SI metric system: \([J] = [N][m]\)
  In the British engineering system: \([\text{ft}][\text{lb}]\). Also \(1 \text{ ft-lb} \approx 1.36 J\).

EXAMPLE 1. How much work is done in lifting your Calculus book (2.1kg) off the floor to put it on a desk that is 0.6m high.

- Case 2: non constant force. (It requires integration.)

Finally, \( W = \lim_{\|P\| \to 0} \sum_{k=1}^{n} F(x_k^*) \Delta x_k \) where \( \|P\| = \max_{k} \Delta x_k \). Thus, work done in moving an object from \( x = a \) to \( x = b \) is
  \[
  W = \int_{a}^{b} F(x) \, dx
  \]

EXAMPLE 2. When a particle is at distance \( x \) feet from the origin, a force of \( 3x^2 + 2x \) pounds acts on it. How much work is done in moving it from \( x = 1 \) to \( x = 3 \) along the \( x \)-axis?
EXAMPLE 3. A spring has a natural length of 1m. If a 50N force is required to keep it stretched to a length 3m, how much work is done in stretching the spring from 2m to 5m?

Solution By Hooke’s law the force required to stretch a spring $x$ units beyond its natural length is

EXAMPLE 4. If the work required to stretch a spring 1ft beyond its natural length is 12ft-lb, how much work is needed to stretch it 9 inches beyond its natural length?
EXAMPLE 5. An aquarium has a form of a cube whose side is 1m. If the aquarium is full of water, find the work needed to pump 50% of the water out of the aquarium. (The density of water is 1000kg/m³.)

EXAMPLE 6. Work the previous example assuming the aquarium is only 1/2 full.
EXAMPLE 7. A tank has a shape of an inverted circular cone with height 10m and base radius 5m. It is filled with water to a height of 8m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000kg/m³.)
EXAMPLE 8. A heavy rope 40 ft long, weighs 0.4 lb/ft and hangs over the edge of a building 120 ft high. How much work is done in pulling the rope to the top of the building?

EXAMPLE 9. A uniform cable hanging over the edge of a tall building is 20 ft long and weighs 30 lb. How much work is required to pull 5 ft of the cable to the top?

REMARK 10. The exact height of the building doesn’t matter.