11.1: Three-dimensional Coordinate System

The three-dimensional coordinate system consists of the origin $O$ and the coordinate axes: $x$-axis, $y$-axis, $z$-axis. The coordinate axes determine 3 coordinate planes: the $xy$-plane, the $xz$-plane and $yz$-plane. The coordinate planes divide space into 8 parts, called octants.
Representation of point \( P(a, b, c) \) and its projections on the coordinate planes:

- \((0, b, c)\) projection of \( P \) onto the \( yz \)-plane
  - \( xy \)-plane \( \Rightarrow z = 0 \)
  - \( yz \)-plane \( \Rightarrow x = 0 \)
  - \( xz \)-plane \( \Rightarrow y = 0 \)

\((a, 0, c)\) projection of \( P \) onto the \( xz \)-plane

\((a, b, 0)\) is projection of \( P \) onto the \( xy \)-plane
Example. Graph the following regions:

(a) \( x = 4 \) in \( \mathbb{R}, \mathbb{R}^2, \mathbb{R}^3 \)

(b) \( x^2 + y^2 = 1 \) in \( \mathbb{R}^2, \mathbb{R}^3 \).
• **Distance formula in** $\mathbb{R}^3$: The distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$ 

**EXAMPLE 1.** *Find an equation of a sphere with radius $r$ and center*

(a) $O(0, 0, 0)$;

|OA| = r

$$\sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2} = r$$

$$x^2 + y^2 + z^2 = r^2$$

(b) $P(a, b, c)$.

|PA| = r

$$\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r$$

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$
EXAMPLE 2. Show that the equation \( x^2 + y^2 + z^2 + x - 2y + 6z - 2 = 0 \) represents a sphere, and find its center and radius.

Complete square \((a \pm b)^2 = a^2 \pm 2ab + b^2\)

\[
\left( x + \frac{1}{2} \right)^2 + \left( y - 1 \right)^2 + \left( z + 3 \right)^2 = 2
\]

\[
\left( x + \frac{1}{2} \right)^2 + \left( y - 1 \right)^2 + \left( z + 3 \right)^2 = 2 + \frac{1}{4} + 1 + 9 = 12 + \frac{1}{4} - \frac{9}{4}
\]

We have equation of sphere centered at \((-\frac{1}{2}, 1, -3)\) with \( r = \sqrt{\frac{49}{4}} = \frac{7}{2} \)