13.4: Polar Coordinates

**REVIEW:**

The connection between polar and Cartesian coordinates:

\[
\begin{align*}
\cos \theta &= \frac{x}{r} & \sin \theta &= \frac{y}{r} \\
x &= r \cos \theta & y &= r \sin \theta \\
r^2 &= x^2 + y^2 & r &= \sqrt{x^2 + y^2} \\
\tan \theta &= \frac{y}{x}
\end{align*}
\]

**REMARK 1.** In converting from the Cartesian to polar coordinates we must choose \( \theta \) so that the point \((r, \theta)\) lies in the correct quadrant.

\[r \geq 0 \quad 0 \leq \theta \leq 2\pi\]
EXAMPLE 2. What curve is represented by the polar equation

(a) $r = 12$

$\sqrt{x^2 + y^2} = 12 \Rightarrow x^2 + y^2 = 12^2$

circle centered at origin with radius 12

(b) $\theta = \frac{\pi}{3}$

ray which makes angle $\frac{\pi}{3}$ with positive direction of polar axis
EXAMPLE 3. Sketch the region in the Cartesian plane consisting of points whose polar coordinates satisfy the following conditions: $1 \leq r \leq 2, \quad \pi/4 \leq \theta \leq \pi$.

$r = 1 \implies x^2 + y^2 = 1$

$r = 2 \implies x^2 + y^2 = 2^2$

Compare with

\[
\begin{array}{c}
\theta = \frac{\pi}{4} \\
\theta = \frac{\pi}{4}
\end{array}
\]
EXAMPLE 4. Find a polar equation for the curve represented by the given Cartesian equation:

(a) $x^2 + y^2 = 2by$

$\begin{align*}
&\begin{cases}
\chi = r \cos \theta \\
y = r \sin \theta
\end{cases} \\
&\begin{cases}
\chi^2 + \chi y = r^2 \\
y^2 = r^2 \sin^2 \theta
\end{cases}
\end{align*}$

\[ r^2 = 2b \chi \sin \theta \]
\[ r = 2b \sin \theta \]

Note that this is a circle

\[
\frac{x^2 + y^2 - 2by + b^2 - b^2}{(y-b)^2} = 0
\]

\[
x^2 + (y-b)^2 = b^2
\]

(b) $(x - a)^2 + y^2 = a^2$

\[
x^2 - 2ax + a^2 + y^2 = a^2
\]

\[
x^2 + y^2 = 2ax \implies r^2 = 2ar \cos \theta \implies r = 2a \cos \theta
\]