QUIZ 3 KEY  Directions: Write the solution in the space provided; full credit will not be given without correct accompanying work. Fully simplify all answers. If possible, give the exact value, or if not possible, give to five decimal places.

1. (15pts) The derivative fails to exist when the graph of the function has a
   a. corner (or sharp peak)
   b. vertical tangent
   c. break (or discontinuity)

2. (15pts) Sketch a possible graph of $y = f(x)$ given the following information.
   \[ f'(x) < 0 \text{ for } x < 4 \]
   \[ f'(x) > 0 \text{ for } x > 4 \]
   \[ f'(x) = 0 \text{ for } x = 4 \]

Given are three possible graphs.
3. (25pts) If \( f(10) = 20 \) and if \( f'(10) = -5 \), estimate \( f(11) \).

\[ f'(11) \approx 20 - 5 = 15 \]

4. The number \( n \) of meters traveled by a particle after \( t \) seconds is \( n(t) = 3t^2 + 5 \).

(5pts) a. What is the total distance traveled during the first 4 seconds?

\[ n(4) - n(0) = (3 \times 4^2 + 5) - (3 \times 0^2 + 5) = 53 - 5 = 48 \text{ meters} \]

(8pts) b. What is the average rate of change in distance during the first 4 seconds?

\[ \frac{n(4) - n(0)}{4 - 0} = \frac{53 - 5}{4} = \frac{48}{4} = 12 \text{ meters per second} \]

(12pts) c. Estimate \( n'(4) \) by using smaller and smaller intervals; use at least three intervals.

[Given are three possible intervals, and there are many others that would be reasonable.]

\[ f'(4) \approx \frac{n(4.01) - n(4)}{4.01 - 4} = 24.03 \text{ meters per second} \]

\[ f'(4) \approx \frac{n(4.001) - n(4)}{4.001 - 4} = 24.003 \text{ meters per second} \]

\[ f'(4) \approx \frac{n(4.0001) - n(4)}{4.0001 - 4} = 24.0003 \text{ meters per second} \]

Therefore \( n'(4) \approx 24 \text{ meters per second} \)