1. (15) State the definitions of the following expressions/terms.

(a) \( \lim_{x \to 2^-} f(x) = \infty \)
This means that for any \( N \) there is a \( \delta \) such that if \( 0 < 2 - x < \delta \), then \( f(x) > N \).

(b) \( f \) is continuous at the point \( x = 3 \).
To say that \( f \) is continuous at \( x = 3 \) means that 3 is in the domain of \( f \) and that \( \lim_{x \to 3} f(x) = f(3) \).

(c) The derivative of \( f \) at the point \( x = 5 \).
\[ f'(5) = \lim_{h \to 0} \frac{f(5 + h) - f(5)}{h}. \]

2. (15) The graph of \( f(x) \) is shown below. Use it to answer the following questions.

(a) \( \lim_{x \to -2^+} f(x) = 1 \)
(b) \( \lim_{x \to -1^-} f(x) = 2 \)
(c) \( \lim_{x \to 1^+} f(x) = +\infty \)
(d) At what points \( a \) in the closed interval \([-2, 2]\) is \( f \) not continuous?
\( f \) is not continuous at the points \( x = -2, 1, \) and 2.
(e) At what points \( a \) in the open interval \((-2, 2)\) does \( \lim_{x \to a} f(x) \) not exist?
The \( \lim_{x \to a} f(x) \) does not exist at the only one point \( a = 1 \).

3. (10) Prove, using the definition, that \( \lim_{x \to 2} (3x - 1) = 5 \).

Let \( \epsilon > 0 \), pick \( \delta = \epsilon/3 \). If \( 0 < |x - 2| < \frac{\epsilon}{3} \), then we must have
\[
|3x - 1 - 5| = |3x - 6| = 3|x - 2| < 3\frac{\epsilon}{3} = \epsilon
\]
4. (15) Evaluate the following limits

(a) \( \lim_{{x \to 1^+}} \frac{|x|}{x} = \lim_{{x \to 1^+}} \frac{|x|}{x} = \frac{1}{1} = 1 \)

(b) \( \lim_{{x \to \infty}} \frac{x^2 - 3x + 1}{10x + 2 - 3x^2} = \lim_{{x \to \infty}} \frac{1 - 3/x + 1/x^2}{10/x + 2/x^2 - 3} = \frac{1}{-3} = \frac{-1}{3} \)

(c) \( \lim_{{x \to 2}} (3x^2 - 6) = 6 \)

5. (20) Let \( f(x) = 3x^2 + 1 \).

(a) Using the definition of the derivative calculate \( f'(2) \).

\[
\begin{align*}
    f'(2) &= \lim_{{h \to 0}} \frac{f(2 + h) - f(2)}{h} \\
    &= \lim_{{h \to 0}} \frac{3(2 + h)^2 + 1 - 13}{h} \\
    &= \lim_{{h \to 0}} \frac{3(4 + 4h + h^2) - 12}{h} \\
    &= \lim_{{h \to 0}} \frac{12h + 3h^2}{h} = \lim_{{h \to 0}} (12 + 3h) \\
    &= 12
\end{align*}
\]

(b) Find the equation for the tangent line to the graph of \( f \) at the point \( (2, 13) \).

\[
\begin{align*}
    \frac{y - 13}{x - 2} &= 12 \\
    y - 13 &= 12(x - 2)
\end{align*}
\]

6. (10) Find a vector equation for the line passing through the two points \((-1, 3)\) and \((-2, 6)\), then find an equation for this same line in the from \( y = f(x) \).

\[
\overrightarrow{r}(t) = \langle -1, 3 \rangle + t(\langle -2, 6 \rangle - \langle -1, 3 \rangle) \\
= \langle -1, 3 \rangle + t(\langle -1, 3 \rangle)
\]

To find a \( y = f(x) \) equation for this line, solve the equation \( x = -1 - t \) for \( t \). This gives \( t = -1 - x \). Now substitute this expression for \( t \) into the equation for \( y \).

\[
\begin{align*}
y &= 3 + 3t \\
   &= 3 + 3(-1 - x) \\
   &= -3x
\end{align*}
\]
7. (15) Let \( \mathbf{r}(t) = (3t^2 - t, t^3 + 1) \) give the position of a particle at time \( t \) in seconds.

(a) Plot this curve for \(-1 \leq t \leq 1\). Be especially careful around \( t = 0 \).

\[ t = 0 \quad (\frac{-1}{12}, 1 + \frac{1}{216}) \quad t = \frac{1}{3} \]

\[ (2, 0) \]

(b) What is the velocity of the particle at \( t = 0 \) and \( t = 1/2 \).

\[ \mathbf{v}(t) = \frac{d}{dt}(3t^2 - t, t^3 + 1) = (6t - 1, 3t^2) \]

\[ \mathbf{v}(0) = (-1, 0) \]

\[ \mathbf{v}(1/2) = (2, 3/4) \]

(c) What is the speed of the particle at \( t = 0 \) and \( t = 1/2 \).

Speed is the magnitude of velocity so

\[ s(t) = \left( (6t - 1)^2 + 9t^4 \right)^{1/2} \]

\[ s(0) = 1 \]

\[ s(1/2) = \left( 4 + \frac{9}{16} \right)^{1/2} = \left( \frac{73}{16} \right)^{1/2} = \frac{\sqrt{73}}{4} \]