1. (6) Use the above axes to plot the bifurcation diagram for the differential equation:

\[ \frac{dy}{dt} = \frac{y}{1+y^2} + \alpha. \]

Be sure to determine the type of each equilibrium points, and all bifurcation values. Justification of your claims is essential for any credit.

2. (4) A 100 gallon tank initially contains 100 gallons of sugar water at a concentration of 0.25 pounds of sugar per gallon. Suppose sugar is added to the tank at a rate of p pounds per minute, sugar water is removed at a rate of 1 gallon per minute, and the water in the tank is kept well mixed.

(a) What value of p should we pick so that, when 5 gallons of sugar solution is left in the tank, the concentration is 0.5 pounds of sugar per gallon?

(b) Is it possible to choose p so that the last drop of water out of the bucket has a concentration of 0.75 pounds of sugar per gallon?