1. A butterfly spread involves either three calls or three puts with different strike prices, and the same expiration date. The position is long one option with the low strike price, long one option with the high strike price and short two options with a strike price halfway between the high and low strike price.

a. Plot the value of a butterfly call spread at expiration as a function of the stock price.

Let $C_h$, $C_m$, and $C_l$ denote the value of the calls with strike prices $X_h$, $X_m$, and $X_l$ respectively. At expiration the portfolio is worth

$$\begin{cases} 
0, & S < X_l \\
S - X_l, & X_l \leq S \leq X_m \\
X_m - S, & X_m \leq S \leq X_h \\
X_h - S, & S \geq X_h 
\end{cases}$$

A plot of this function appears below.

b. Plot the value of a butterfly put spread at expiration as a function of the stock price.

It turns out that the value of a put butterfly spread is the same as that of a call butterfly spread. So we have the same plot.

c. For a butterfly call spread determine the initial cost, maximum profit, and maximum loss. Let $XL$, $XM$, and $XH$ denote the low middle, and high strike prices with $XM : YXL + XH$2. Let $CL$, $CM$, and $CH$ denote the value of these respective calls.

Initial cost equals $C_h + C_l - 2C_m$

Maximum profit equals $\frac{X_h - X_l}{2} - C_h + C_l - 2C_m$

Maximum loss equals $C_h + C_l - 2C_m$
d. For a negative butterfly put spread determine the initial cost, maximum profit, and maximum loss. Let \( XL, XM, \) and \( XH \) denote the low middle, and high strike prices with \( XM = \frac{XL + XH}{2} \). Let \( PL, PM, \) and \( PH \) denote the value of these respective calls. Note: the word negative mean you’ve sold rather than bought a butterfly spread.

Initial cost equals \( 2P_m ? P_h ? P_l \)

Maximum profit equals \( 2P_m ? P_h ? P_l \)

Maximum loss equals \( 2P_m ? P_h ? P_l ? \left( \frac{X_h ? X_l}{2} \right) \)

2. (10) Let \( CH \) and \( CL \) denote the costs of two call options with the same expiration date and respective strike prices \( X_H \); \( X_L \). Determine via an arbitrage argument whether \( CH \); \( C_L \) or \( C_H \); \( C_L \), or if there is any relationship between the two prices.

An arbitrage argument shows that \( C_H \) \textasciitilde \( C_L \). That is, the higher the strike price the lower the cost of the call. This follows from the fact that at expiration the difference of the worth of the two calls is \( CL ? CH : \mathbb{1} ? X_L ? \frac{X_H ? X_L}{2} \).

\( CL \) ? \( CH \) : \[ \begin{align*} 0, & \quad S \leq X_L \\ S \leq X_L, & \quad X_L \leq S \leq X_H \\ X_H ? X_L, & \quad X_H \leq S 
\end{align*} \]

No matter the value of \( S \), at expiration the value of \( CL \) ? \( CH \) is nonnegative. Thus, \( C_H \) \textasciitilde \( C_L \) must be true all the time.

3. (20) Take at least 30 consecutive daily closings of the stock you picked and determine the drift and volatility parameters \( \bar{S} \) and \( \bar{J} \) for a log-normal model of stock prices. Then plot, on the same graph, the stock data and the prices given by your log-normal model.

Once \( S \) and \( J \) have been determined, generate \( Z_i \), where these numbers are independent and generated by a random number generator which has the standard normal distribution. Start with \( S_0 \) and compute \( S_i \) via the following formulas

\( S_1 : e^{\sqrt{\bar{J}^2}Z_1}e^{\bar{S}^2/2}\bar{\sigma}(S_0) \)

\( S_2 : e^{\sqrt{\bar{J}^2}Z_2}e^{\bar{S}^2/2}\bar{\sigma}(S_1) \)

\( S_i : e^{\sqrt{\bar{J}^2}Z_i}e^{\bar{S}^2/2}\bar{\sigma}(S_{i-1}) \)