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Abstract: The talk will describe a recent result by Y. Pinchover (Technion, Israel) and the speaker. Liouville properties of elliptic equations on non-compact Riemannian manifolds have been studied extensively for the past 30 years, since the work done by S. T. Yau. Here by the Liouville property we mean finite dimensionality of the space of solutions of a given polynomial growth (in particular, of the space of bounded solutions). This is a non-generic property in the sense that small variations in the operator's coefficients destroy it, so the study must be operator specific. Major results have been achieved in work by T. Colding and W. Minicozzi, and by P. Li with co-authors. A parallel activity has existed in complex analysis, where one is interested infinite dimensionality of the spaces of holomorphicfunctions of a polynomial growth with respect to a suitable Riemannian metric. We will present the following result: the space of an abelian covering (i.e., a regularcovering with the abelian deck group) over a compact complex manifold has Liouville property. Here on the covering one can use the lift of any Riemannian metric from the base. Such a result was known previously only for special classes of bases, although if one restrcits considerations to bounded functions only, this was known for a wider variety of coverings (e.g., nilpotent ones). The result is proven using that thed-bar operator can be included into an elliptic complex and then applying the technique developed by the authors for elliptic equations.