Problems to Introduction to Real Analysis, (Math446)
Due: 11/23/04

Problem 1. Let \((X, T)\) be a topological space. \(A, B \subset X\)
   a) \(\overline{A \cap B} \subset \overline{A} \cap \overline{B}\),
   b) \(\overline{A \setminus B} \subset \overline{A} \setminus \overline{B}\).

Problem 2. Consider on \(\mathbb{R}\) the topology \(T_h\) generated by all half open
   intervals of the form \([a, b[,\). Denote the usual topology on \(\mathbb{R}\) by \(T_u\).
   a) Show that \(T_h\) at least as fine as \(T_u\) (i.e. show that \(T_u \subset T_h\).
   b) Show that the intervals \([a, b[\) are also closed with respect to \(T_h\).
   c) Does \(\left\{ \frac{1}{n} \right\}\) converge in \(T_h\)? Does \(\left\{ -\frac{1}{n} \right\}\) converge in \(T_h\)? Support your
      answers.
   d) Show that \(\mathbb{R}\) is separable with respect to \(T_h\).

Problem 3. Let \(B\) be a base for \((X, T)\) and let \(B \subset B^* \subset T\). Show that \(B^*\)
   is also a base for \(T\).

Problem 4. The topology generated by a metric space which is separable
   has a base which is countable.

Problem 5. Consider the set of all half planes on \(\mathbb{R}^2\), i.e.
   \[
   B = \bigcup_{a \in \mathbb{R}} \{ (x, y) : y > a \} \cup \{ (x, y) : y < a \} \cup \{ (x, y) : x > a \} \cup \{ (x, y) : x < a \}.
   \]
   Show that \(B\) is a subbasis which generates the usual topology.

Problem 6.* There is no metric \(d\) on \(\mathbb{R}\) so that the topology generated by \(d\)
   equals to the topology \(T_h\) described in Problem 2.
   Hint: Use Problems 2 d and Problem 4.