1. (10 points) Find the inverse of $A = \begin{pmatrix} -1 & 0 & 3 \\ 0 & 1 & 4 \\ 1 & 1 & 0 \end{pmatrix}$.

Use it to solve $XA = \begin{pmatrix} 0 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 0 \end{pmatrix}$. 
2. (10 points) Consider the polynomials

\[ p_1(x) = 1 - x^2 \]
\[ p_2(x) = 2 - x - x^2 \]
\[ p_3(x) = 1 - x \]

and the vector space

\[ W = \text{Span}(p_1, p_2, p_3). \]

Find a subset of \{p_1, p_2, p_3\} which is a basis for \( W \). Prove it spans \( W \) and is linearly independent.
3. Consider the vector space $P_3$, the set of polynomials of degree 3 or less?
   • (5 points) Scantron #1 Which of the following is NOT a subspace of $P_3$?
     a. $A = \{ p \in P_3 \mid p(0) = 0 \}$
     b. $B = \{ p \in P_3 \mid p(1) = 0 \}$
     c. $C = \{ p \in P_3 \mid p(0) = p(1) \}$
     d. $D = \{ p \in P_3 \mid p(0) + p(1) = 0 \}$
     e. $E = \{ p \in P_3 \mid p(0) = 1 \}$

4. Consider the vector space $\mathbb{R}^+$ of all positive real numbers with the operations of
   Vector Addition: $x \oplus y = xy$ (real number addition)
   Scalar Multiplication: $a \cdot x = x^a$ (real number exponentiation)
   • (5 points) Scantron #2 Translate the vector identity
     $0 \cdot x = 0$
     into ordinary arithmetic.
     a. $1^x = 1$
     b. $x^0 = 1$
     c. $0^x = 0$
     d. $x^1 = x$
     e. $0^x = 1$
5. Consider the linear map $L : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ given by $L(\vec{x}) = A\vec{x}$ where $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 4 \\ 3 & -1 & 4 \end{pmatrix}$.

- (10 points) Solve $L(\vec{x}) = \begin{pmatrix} 2 \\ -1 \\ 2 \\ 4 \end{pmatrix}$.

- (5 points) Scantron #3 Describe the solution set:
  a. No Solutions
  b. Unique Solution (Point in $\mathbb{R}^3$)
  c. $\infty$-Many Solutions (Line in $\mathbb{R}^3$)
  d. $\infty$-Many Solutions (Plane in $\mathbb{R}^3$)
  e. $\infty$-Many Solutions (All of $\mathbb{R}^3$)

- (5 points) Scantron #4 Is $L$ a one-to-one function?
  a. Yes
  b. No
6. Again consider the linear map \( L : \mathbb{R}^3 \to \mathbb{R}^4 \) given by \( L(\vec{x}) = A\vec{x} \) where 
\[
A = \begin{pmatrix}
1 & -1 & 0 \\
0 & 1 & 2 \\
2 & 0 & 4 \\
3 & -1 & 4
\end{pmatrix}.
\]

- (10 points) Solve \( L(\vec{x}) = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \).

- (5 points) Scantron #5 Describe the solution set:
  a. No Solutions
  b. Unique Solution (Point in \( \mathbb{R}^3 \))
  c. \( \infty \)-Many Solutions (Line in \( \mathbb{R}^3 \))
  d. \( \infty \)-Many Solutions (Plane in \( \mathbb{R}^3 \))
  e. \( \infty \)-Many Solutions (All of \( \mathbb{R}^3 \))

- (5 points) Scantron #6 Is \( L \) an onto function?
  a. Yes
  b. No
7. Again consider the linear map \( L : \mathbb{R}^3 \rightarrow \mathbb{R}^4 \) given by \( L(\vec{x}) = A\vec{x} \) where \( A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 4 \\ 3 & -1 & 4 \end{pmatrix} \).

- (5 points) Find \( \text{Ker}(L) \), the kernel (or null space) of \( L \).

- (5 points) Give a basis for \( \text{Ker}(L) \). (No proof)

- (5 points) What is the dimension of \( \text{Ker}(L) \)? (No proof)
8. Again consider the linear map \( L : \mathbb{R}^3 \to \mathbb{R}^4 \) given by \( L(\vec{x}) = A\vec{x} \) where \[ A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 4 \\ 3 & -1 & 4 \end{pmatrix}. \]

- (5 points) Find \( \text{Im}(L) \), the image (or range) of \( L \).

- (5 points) Give a basis for \( \text{Im}(L) \). (No proof)

- (5 points) What is the dimension of \( \text{Im}(L) \)? (No proof)