TEXAS A&M UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 308-506

Exam 1.A, 21 Sep 2005

Name:	**Mark:** /40

1. Sketch the direction field for the equation

$$\frac{dx}{dt} = 5x(x-1)^2(2-x)$$

in the region $0 \le t \le 1$, $0 \le x \le 2.5$. What is the $t \to \infty$ limit of the solution satisfying $x(0) = 1.5$? Can the solution satisfying $x(0) = 0.5$ ever grow to 1.5? Justify.

(8 marks)

2. Solve the IVP

$$\frac{dx}{dt} = x^2(1 + \sin(t)), \qquad x(0) = 1.$$

(8 marks)

3. Solve the IVP

$$\cos(x)\frac{dy}{dx} + 2\sin(x)y = x\cos^3(x), \qquad y(0) = 2.$$

(8 marks)

4. The body of a murder victim was discovered at 6pm. Police officers measured the body temperature at 6.10pm and then again at 7.10pm; the temperature readings were 29^{o}C and 25^{o}C correspondingly. The temperature of the building is maintained by an air conditioning system at the constant 21^{o}C. Assuming the victim had the normal temperature of 37^{o}C at the time of the murder, what time did the murder happen? Use Newton's law of cooling

$$\frac{dT}{dt} = k(M - T),$$

where T is the temperature of the body, M is the temperature of the environment and k is a proportionality coefficient. Write down the equations you are solving. (Hint: take 6.10pm as $t = 0$).

(8 marks)

5. A sailboat has been running (on a straight course) under a light wind at 1 m/sec. Suddenly the wind picks up, blowing hard enough to apply a constant force of 600 N to the boat. The only other force on the boat is water resistance that is proportional to the velocity of the boat with the proportionality constant $b = 100$ N-sec/m. If the mass of the boat is 150 kg, find the velocity of the boat as a function of t. What is the limiting velocity of the boat.

(8 marks)