Engineering Mathematics II

Honors Sections 819–820

Here are two problems to analyze using Maple and using your wits.

1. The *greatest integer function* (also called the *floor function*), denoted $[x]$ or $\lfloor x \rfloor$, is the greatest integer less than or equal to x. For instance, $\lfloor 4.2 \rfloor = 4$, $\lfloor 7 \rfloor = 7$, and $\lfloor -2.2 \rfloor = -3$. Maple knows this function by the name "`floor`". Consider the infinite series

$$\sum_{n=1}^{\infty} \left(\frac{1}{\lfloor \sqrt{n} \rfloor} \right)^3 = 1 + 1 + 1 + \frac{1}{8} + \frac{1}{8} + \cdots .$$

   (a) Give Maple the command

```
evalf(sum(1/floor(sqrt(n))^3, n=1..4));
```

   and see what happens. Then give Maple the command

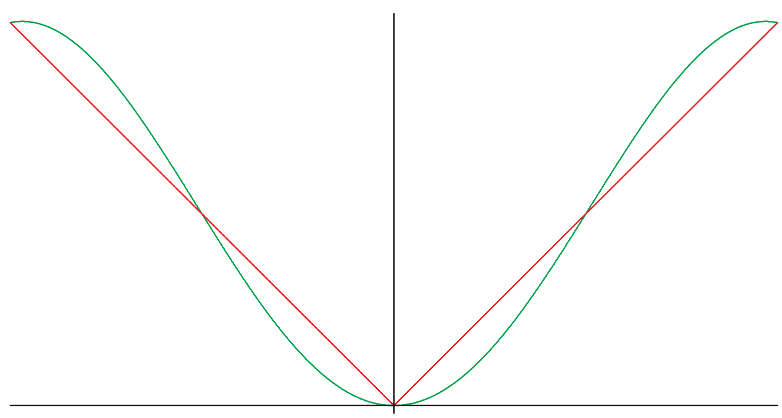
```
evalf(sum(1/floor(sqrt(n))^3, n=1..infinity));
```

   and see what happens. Can you explain the results?

   (b) This series converges. Can you prove it?

   (c) See how accurate a value you can get for the sum of the series. (The value correct to six decimal places is 4.491925.)

2. See how closely you can approximate the graph of the absolute-value function $|x|$ by the graph of a *polynomial*. For example, here is a plot of the graph of $|x|$ together with the polynomial $\frac{14}{45}x^6 - \frac{31}{18}x^4 + \frac{217}{90}x^2$.



Can you do better?