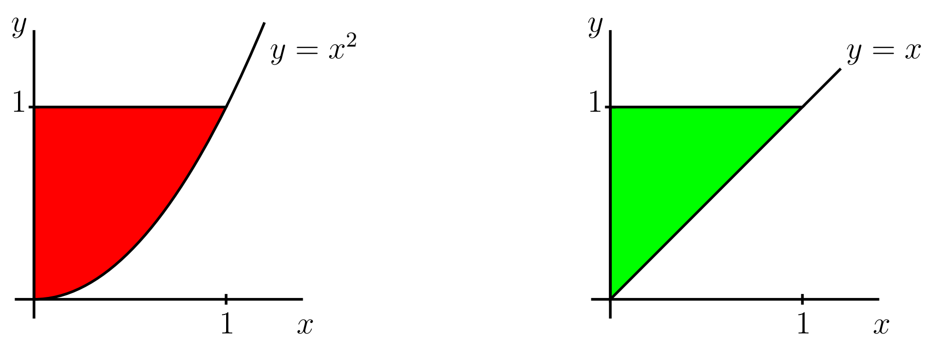
Math 152 **Engineering Mathematics II** Fall 2004

Honors Sections 201–202

First Examination

Work all five problems. These are essay questions. To obtain maximal credit, show your work and explain your reasoning.

1. In a book titled *On conoids and spheroids*, Archimedes of Syracuse (who lived in the third century B.C.) proved that the volume of a paraboloid of revolution is 3/2 the volume of a cone with the same vertex, axis, and base. Verify this result of Archimedes by computing, for each of the two regions illustrated in the figure, the volume of the solid obtained by revolving the region around the y-axis.



2. A hot tub has a constant horizontal cross section in the shape of Texas. When the hot tub is *half full* of water, the work required to pump the water out over the top is 12,000 joules. Determine how much work is required to pump the water out when the hot tub is *full* of water.

3. In the theory of Fourier series, one needs the *orthogonality relation*

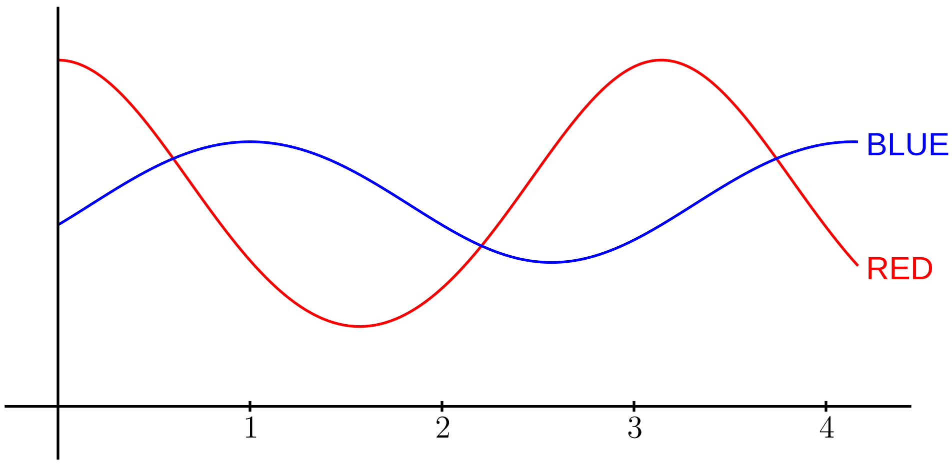
$$\int_0^{2\pi} \sin(nx)\cos(mx)\,dx = 0 \qquad \text{for all integers } n \text{ and } m.$$

Derive this equation.

4. The figure shows the graph of a certain function f and the graph of a second function g that is defined in terms of f as follows:

$$g(s) = \frac{1}{2}\int_{s-2}^{s} f(t)\,dt.$$

In the language of finance, the function g is a *moving average* of the function f.



In the figure, which curve represents f and which curve represents g? Explain your reasoning.

5. Scheherazade (or Shahrazad) is the heroine of the *Arabian Nights' Entertainment*, also known as *The Thousand and One Nights*. She might have used the following problem to interest King Shahriyar, had he known the method of partial fractions.

Suppose a_0, a_1, ..., a_{1000} are constants such that

$$\begin{aligned}\int \frac{a_0 + a_1x + a_2x^2 + \cdots + a_{1000}x^{1000}}{x(x+1)(x+2)\dots(x+1000)}\,dx \\ = 1001\ln|x| + 1000\ln|x+1| + 999\ln|x+2| \\ + \cdots + 2\ln|x+999| + \ln|x+1000| + C.\end{aligned}$$

Determine the value of a_0.