

Work all five problems. These are essay questions. To obtain maximal credit, show your work and explain your reasoning.

1. Atmospheric pressure varies with the weather and with the position on the globe. For reference purposes, the international *standard atmosphere* is defined to be 101,325 Pa. (Recall that $1 \text{ Pa} = 1 \text{ N/m}^2$.)

The mathematician Evangelista Torricelli (shown here on an Italian postage stamp) conceived the idea of the barometer in 1643 and found by experiment that atmospheric pressure is equal to the pressure at the bottom of a column of mercury 0.76 m high.

Determine the value of the ratio $\frac{(\text{density of mercury})}{(\text{density of water})}$.



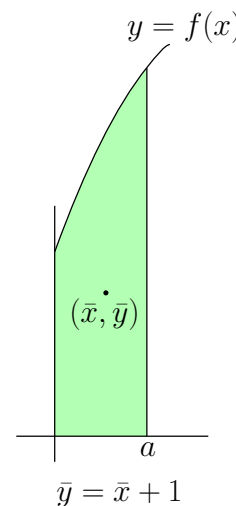
2. You know how to compute the arc length of a curve in the plane by integration. This problem asks you to generalize to the case of a curve in three-dimensional space by using the arc length element

$$ds = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}.$$

Find the length of the *helix* that is given in parametric form by the equations

$$\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(t) \\ z(t) = t \end{cases} \quad \text{for } 0 \leq t \leq 6\pi.$$

3. Find a function f with the property that for every positive number a , the centroid of the region in the first quadrant under the graph of f between 0 and a has a y -coordinate that is exactly one unit bigger than the x -coordinate.



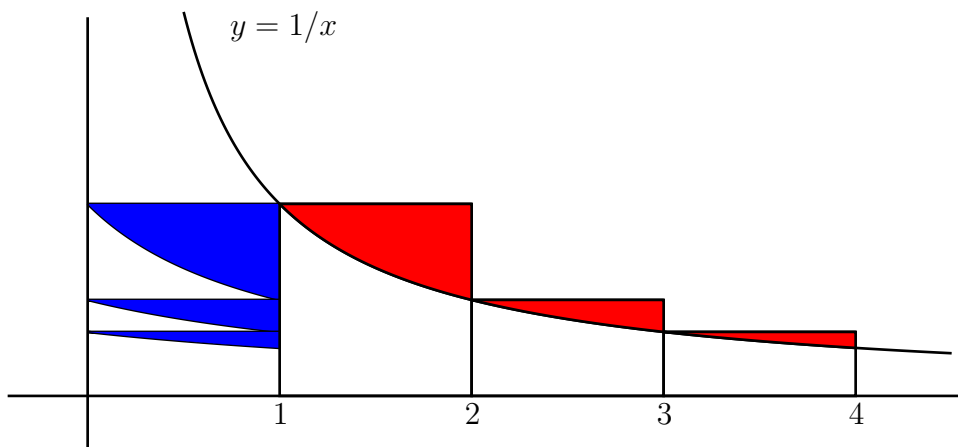
4. Solve the differential equation $\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x}$.

Hint: Introduce a new dependent variable u via $u = y/x$. You should be able to transform the given equation into a separable differential equation (in the variables u and x) that you know how to solve.

5. You know that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. This problem quantifies the rate of growth of the partial sums of the harmonic series. By suitably interpreting the figure below, show that the limit

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \log(n+1) \right)$$

- (a) exists, and
 (b) has a value that is between 0 and 1.



(The value of the limit is known as *Euler's constant*, usually denoted by the Greek letter γ . The value of γ is approximately 0.577.)