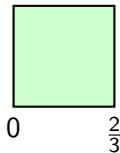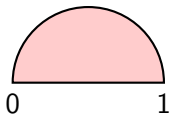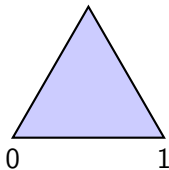Math $3^2 \times 19$
Section $2^9 - 9$
Harold P. Boas

Two warm-up puzzles

1. Which figure has the largest area?

0 1 0 1 0 $\frac{2}{3}$

2. If $f(x) = \dfrac{1}{1 - x}$, find the composite function $f \circ f \circ f$.

Answers: The square has the largest area, and $f \circ f \circ f(x) = x$.

Vectors

The Irish mathematician William Rowan Hamilton (1805–1865) introduced the words *vector* and *scalar* in mathematics.

A vector has both a magnitude and a direction.

A scalar is just a number, without a direction.

Examples

- ► velocity
- ► force

Notation for vectors

The textbook denotes vectors with boldface letters, like **v**.
For writing by hand, a better notation is $\vec{v}$.

A two-dimensional vector $\vec{v}$ has two *components* v_1 and v_2 representing horizontal and vertical displacements.

The textbook uses angle brackets to represent a vector in terms of components: $\vec{v} = \langle v_1, v_2 \rangle$.

Example: The vector $\langle 2, 3 \rangle$ represents a motion of 2 units to the right and 3 units up.

Magnitude (length)

If $\vec{v} = \langle v_1, v_2 \rangle$, then the magnitude of $\vec{v}$ is given by the Pythagorean law as $\sqrt{v_1^2 + v_2^2}$.

Notation for magnitude: either $|\vec{v}|$ or $\|\vec{v}\|$.

Example: If $\vec{v} = \langle 3, 4 \rangle$, then $|\vec{v}| = 5$.

Standard basis vectors

Traditionally, **i** or $\vec{\imath}$ means the vector $\langle 1, 0 \rangle$, and **j** or $\vec{\jmath}$ means $\langle 0, 1 \rangle$.

So $\langle 2, 3 \rangle$ and $2\vec{\imath} + 3\vec{\jmath}$ mean the same thing.

The algebra of vectors

Example: If $\vec{v} = 2\vec{\imath} + 3\vec{\jmath}$, and $\vec{w} = 4\vec{\imath} - \vec{\jmath}$, then $5\vec{v} - 2\vec{w} = 2\vec{\imath} + 17\vec{\jmath}$.

Vector addition is commutative and associative, and scalar multiplication distributes over vector addition.

Assignment

In section J.1 do odd-numbered problems 1–15 and check your
answers in Appendix L.
(not to hand in)