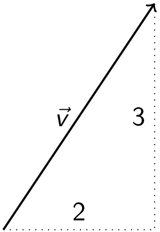
Recap from yesterday

A vector has a magnitude and a direction.

Example

If $\vec{v} = \langle 2, 3 \rangle = 2\vec{\imath} + 3\vec{\jmath}$, then $|\vec{v}| = \sqrt{2^2 + 3^2} = \sqrt{13}$.



Unit vectors, parallel vectors

If $|\vec{v}| = 1$, then $\vec{v}$ is a *unit vector*.

The vector $\dfrac{\vec{v}}{|\vec{v}|}$ is a unit vector that points in the same direction as $\vec{v}$.

In general, vectors $\vec{v}$ and $\vec{w}$ are *parallel* if they point in the same direction.

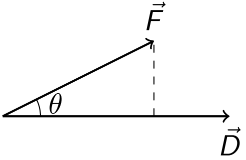
Work done by a force

Physics says that

$$\text{work} = \text{force times displacement}$$

when the force and the displacement are in the same direction.

In general, only the component of the force in the direction of the displacement contributes to the work.



$$\text{Work} = |\vec{F}|\,|\vec{D}|\cos(\theta).$$

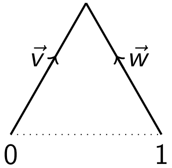
Dot product (also called scalar product)

The formula for work motivates an operation that takes two vectors as input and produces a scalar as output:

$$\vec{v} \cdot \vec{w} \overset{\text{definition}}{=} |\vec{v}|\,|\vec{w}|\,\cos(\theta),$$

where θ is the angle between the two vectors.

Example



$$\vec{v} \cdot \vec{w} = 1/2$$

A component formula for the dot product

If $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$, then

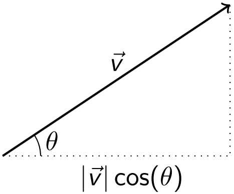
$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2.$$

Example

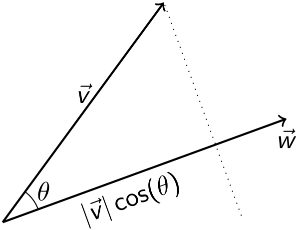
If $\vec{v} = \langle 3, 3 \rangle$ and $\vec{w} = \langle 2, 0 \rangle$, what is the angle between $\vec{v}$ and $\vec{w}$?

Answer: $\pi/4$ radians

Projection



The $\vec{\imath}$ component of $\vec{v}$ equals $|\vec{v}|\cos(\theta)$, or $\vec{v} \cdot \vec{\imath}$.



The component of $\vec{v}$ in the direction of $\vec{w}$ equals $|\vec{v}|\cos(\theta)$, or $\vec{v} \cdot \dfrac{\vec{w}}{|\vec{w}|}$ (the dot product of $\vec{v}$ with a *unit* vector in the direction of $\vec{w}$).

Assignment

- Do the odd-numbered problems 1–11 in Appendix J.2 and check your answers in Appendix L (not to hand in).
- Be prepared for a quiz tomorrow (Thursday) on what we covered yesterday and today.