How do you spell the word “asymptote”?

New limits from old

Suppose $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 2} g(x) = 2$.

- $\lim_{x\to 2} \big(f(x) + g(x)\big) = 3 + 2 = 5.$
- $\lim_{x\to 2} f(x)g(x) = 2 \times 3 = 6.$
- $\lim_{x\to 2} f(g(x)) = 3.$
  $f(g(x))$ can also be written as $(f \circ g)(x)$.
- $\lim_{x\to 2} (g \circ f)(x) =$ not enough information to answer this question

A subtlety

Is it always true that $\lim_{x\to 0}(f(x)+g(x)) = \lim_{x\to 0} f(x) + \lim_{x\to 0} g(x)$?

Examples

- If $f(x) = \dfrac{1}{x^2}$ and $g(x) = 1 - \dfrac{1}{x^2}$,then the left-hand side is 1, but the right-hand side is $\infty - \infty$.
- If $f(x) = \operatorname{sgn}(x) := \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$
  and $g(x) = -f(x)$, then the left-hand side is 0,
  but the right hand side is
  does not exist + does not exist.

The equality *is* true when the two limits on the right-hand side exist and are finite.

Squeeze theorem (or sandwich theorem)

Example

$\lim_{x \to 0} x \cos(1/x) = ?$

Solution. Since $-|x| \leq x \cos(1/x) \leq |x|$, and $\lim_{x \to 0} |x| = 0$ and $\lim_{x \to 0} (-|x|) = 0$, it must be that $\lim_{x \to 0} x \cos(1/x) = 0$.

Assignment (not to hand in)

Solve the following problems and check your answers in the book.

- In Section 2.2, problems 35, 37, 41.
- In Section 2.3, problems 17, 19, 25, 37, 49, 51, 59, 65.