Exam Results

- Scoring algorithm: 30 + (10 points per problem).
- Class statistics: mean 82, median 83, maximum 98. Good job!
- Solutions are posted.

Exam follow-up

- What is the difference between "zero slope" and "no slope"? No slope corresponds to a vertical line; zero slope corresponds to a horizontal line.
- Absolute value can be interpreted geometrically as distance: $|x-2|$ means the distance on the number line between x and 2. So $|x-2|$ and $|2-x|$ mean the same thing.

  For a vector, $|\vec{v}|$ is the distance between the tail of the vector and the head of the vector, hence $\sqrt{v_1^2 + v_2^2}$.

Some consequences of $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

- $\frac{d}{dx} e^x = e^x$
- $\frac{d}{dx} \sin(x) = \cos(x)$
- $\frac{d}{dx} \cos(x) = -\sin(x)$
- $\frac{d}{dx} x^n = nx^{n-1}$

Derivatives and algebra

Limits preserve sums, so derivatives do too:

$$\begin{aligned}\frac{d}{dx}\left(2e^x + 3\sin(x)\right) &= \frac{d}{dx}(2e^x) + \frac{d}{dx}(3\sin(x)) \\ &= 2e^x + 3\cos(x).\end{aligned}$$

But *products* are a different story:

$$\frac{e^{x+h}\sin(x+h) - e^x\sin(x)}{h} \neq \left(\frac{e^{x+h} - e^x}{h}\right)\left(\frac{\sin(x+h) - \sin(x)}{h}\right).$$

⚠ The derivative of a product is *not* equal to the product of the derivatives!

The product rule for derivatives

$$\boxed{(fg)' = f\,g' + f'g} \qquad \text{(if } f \text{ and } g \text{ are differentiable)}$$

Example

$$\frac{d}{dx}\left(e^x \sin(x)\right) = e^x \frac{d}{dx}\sin(x) + \left(\frac{d}{dx}e^x\right)\sin(x) = e^x\cos(x) + e^x \sin(x)$$

The quotient rule for derivatives

⚠ The derivative of a quotient is not equal to the quotient of the derivatives.

$$\left(\frac{f}{g}\right)' = \frac{g\,f' - f\,g'}{g^2}$$

Example

$$\frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{(\cos(x))^2} = \frac{1}{(\cos(x))^2}$$

Thus $\dfrac{d}{dx}\tan(x) = (\sec(x))^2 = 1 + (\tan(x))^2$.

The chain rule

How do the graphs of $\sin(x)$ and $\sin(2x)$ compare?

The graph of $\sin(2x)$ is compressed by a factor of 2, so the graph changes twice as fast.

Consequently, $\frac{d}{dx}\sin(2x) = 2\cos(2x)$

Similarly, the derivative of $\sin(g(x))$ equals $\cos(g(x))$ times $g'(x)$.

Assignment

- Section 3.1, Exercises 3, 7, 15, 33, 37, 55.
- Section 3.2, Exercises 3, 7, 23, 29, 43, 49, 55.
- Section 3.3, Exercises 3, 7, 9, 11, 13, 17, 23.