Application of implicit differentiation to inverse functions

Example

If $f(x) = \ln(x)$ (the "natural" logarithm), what is $f'(x)$?

Solution

The natural logarithm is the inverse of the exponential function:
$e^{\ln(x)} = x$ and $\ln(e^x) = x$
So $e^{f(x)} = x$. Differentiate using the chain rule:
$e^{f(x)} f'(x) = 1$
Therefore $f'(x) = \dfrac{1}{e^{f(x)}} = \dfrac{1}{x}$

$$\boxed{\frac{d}{dx}\ln(x) = \frac{1}{x}}$$

Tangents to parametric curves

Example

Suppose a curve is described in parametric form:

$$\begin{aligned} & x(t) = 2t - \ln(1+t^2), \\ & y(t) = e^t \sin(t), \\ \text{or} \qquad & \vec{r}(t) = \left(2t - \ln(1+t^2)\right)\vec{\imath} + \left(e^t \sin(t)\right)\vec{\jmath}. \end{aligned}$$

Find a unit vector tangent to the curve at the point where $t = 0$.

Solution

$\vec{r}\,'(0)$ is a tangent vector. Compute the derivative to see that $\vec{r}\,'(0) = 2\vec{\imath} + \vec{\jmath}$. A corresponding unit vector is $\frac{2}{\sqrt{5}}\vec{\imath} + \frac{1}{\sqrt{5}}\vec{\jmath}$.

The slope of the tangent line is $1/2$.

Assignment (not to hand in)

- Appendix K.1, Exercises 11, 13, 15, 17, 21, 23.
- Appendix K.2, Exercises 3, 5, 7, 9, 13, 17, 21.