Some terminology and notation: differentials

The symbol Δ (Greek letter Delta) denotes change:
Δx is a change in x, and Δy is a change in y.

The ratio $\dfrac{\Delta y}{\Delta x}$ is an *average* rate of change.

$\dfrac{dy}{dx} = \lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x}$ is an *instantaneous* rate of change.

For the input variable, Δx and dx mean the same thing.

For the output variable, Δy means the exact change, but dy means the approximate change given by the tangent-line approximation:
$dy = f'(x)\,dx$.

Example

Exercise 16 in Section 3.10

(a) Find the differential dy when $y = \cos(\pi x)$.

$dy = -\pi \sin(\pi x)\, dx$

(b) Evaluate dy when $x = \frac{1}{3}$ and $dx = -0.02$.

Quadratic approximation

The linear approximation corresponds to a line tangent to a curve.

The quadratic approximation corresponds to a parabola that has the same slope as the curve and the same second derivative.

$$f(x) \approx f(b) + f'(b)(x-b) + \frac{1}{2}f''(b)(x-b)^2$$

Example

Suppose $f(x) = e^x$ and $b = 0$. The quadratic approximation says $e^x \approx 1 + x + \frac{1}{2}x^2$ when x is close to 0.
For instance, $e^{0.1} \approx 1 + 0.1 + \frac{1}{2}(0.1)^2 = 1.105$

Assignment

Travel safely over Spring Break.