Reminder

- The Math Department drop-in Help Session for Math 151/171 takes place in Blocker 117 on Tuesday, Wednesday, and Thursday evenings, 5:00–7:30; and in Blocker 150 on Monday evenings, 7:30–10:00.
- I have office hours 2:00–3:00 on Monday and Wednesday afternoons in Blocker 601L. I am available also by appointment.
- Our teaching assistant, Angelique, has office hours in Blocker 221B on Tuesday and Thursday afternoons 1:00–2:00 and on Wednesday afternoons 3:00–4:00.

About the exam

- The second exam takes place in class Thursday (March 28).
- Please bring your own paper to the exam.
- Main topics:
  - chain rule, product rule, quotient rule
  - implicit differentiation
  - tangents to parametric curves
  - related rates
  - linear approximation
  - extreme values
  - derivatives and the shape of graphs
  - theorems: existence of extrema; Fermat's theorem; Rolle's theorem; the mean-value theorem

Testing critical numbers

If $f'(c) = 0$, how can you tell if there is a *maximum* at c or a *minimum* at c?

Second-derivative test

If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.
If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.
If $f'(c) = 0$ and $f''(c) = 0$, then try something else.

First-derivative test

If $f'(c) = 0$ and the sign of f' changes from negative to positive, then f has a local minimum at c.
If $f'(c) = 0$ and the sign of f' changes from positive to negative, then f has a local maximum at c.
If $f'(c) = 0$ but the sign of f' does not change, then f has a "saddle point" (neither a local max nor a local min).

What goes up must come down

Theorem (Rolle's theorem)

If f is differentiable on an interval, and $f(a) = f(b)$, then there is some number c between a and b for which $f'(c) = 0$.

Tipsy Rolle's theorem

Theorem (Mean-value theorem)

If f is differentiable on an interval, then the average rate of change $\frac{f(b)-f(a)}{b-a}$ equals the instantaneous rate change $f'(c)$ at some number c between a and b.

Example application of the mean-value theorem

Problem

Prove that $\left|\sin(x) - \sin(y)\right| \le |x - y|$ for all real numbers x and y.

Solution

By the mean-value theorem, there exists a number c for which $\sin(x) - \sin(y) = \cos(c)(x - y)$. Then

$$\left|\sin(x) - \sin(y)\right| = \left|\cos(c)\right| |x - y| \le |x - y|.$$