Announcement

Freshman Sophomore Math Contest

7:00–9:00 pm, Tuesday, April 9

Blocker 220

Antiderivatives

If $F' = f$, then f is the derivative of F, and F is an *antiderivative* of f.

Example

If $f(x) = \sin(x) + x\cos(x)$, then an antiderivative of $f(x)$ is $x\sin(x)$ because $\dfrac{d}{dx} x\sin(x) = \sin(x) + x\cos(x)$ by the product rule.

Another antiderivative of $f(x)$ is $171 + x\sin(x)$.

In general, if $F(x)$ is one antiderivative of $f(x)$, then $F(x) + \text{constant}$ is another antiderivative.

So an antiderivative is usually written as $F(x) + C$.

Notation

An antiderivative is also called an *indefinite integral* [coming up in Section 5.4].

The standard notation is $\int f(x)\,dx = F(x) + C$.

Example

$\int 7x^6\,dx = x^7 + C$.

Power rule

For derivatives: $\frac{d}{dx}x^n = nx^{n-1}$

For antiderivatives: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ as long as $n+1 \neq 0$.

Special case: $\int x^{-1}\, dx = \int \frac{1}{x}\, dx = \ln(x) + C$ when $x > 0$.

Even more special case: $\int x^{-1}\, dx = \ln|x| + C$ when $x < 0$.

Max–min word problems

Strategy:

- Name the variables.
- Identify the function to be extremized.
- Use any constraint equations to write the function in terms of just one variable.
- Find critical numbers and check the endpoints.

Quiz

1. Find $f(x)$ if $f''(x) = 8x^3 + 5$, $f(1) = 0$, and $f'(1) = 8$. [Section 4.9 #40]

2. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$. [Section 4.7 #22]

Assignment (not to hand in)

- Section 4.7 [optimization word problems]: Exercises 3, 5, 7, 13, 15, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 65, 73
- Section 4.9 [antiderivatives]: Exercises 1, 5, 7, 9, 13, 15, 17, 21, 25, 35, 41, 47, 51, 61, 75