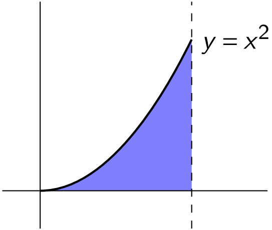
Area

Example

How can the area under a parabola be determined?



Idea from the ancient Greeks: Approximate the area under a curve by a sum of areas of rectangles.

Notation for sums

$\sum_{i=1}^{n} i$ means $1+2+\cdots+n$, which equals $\frac{n(n+1)}{2}$.

$\sum_{i=1}^{n} i^2$ means $1^2+2^2+\cdots+n^2$, which equals $\frac{n(n+1)(2n+1)}{6}$.

$\sum_{i=1}^{n} 2^i$ means $2+2^2+\cdots+2^n$, which equals $2^{n+1}-2$.

Notation for "Riemann sums"

Suppose $f(x)$ is defined when $a \le x \le b$.

Subdivide the interval $[a, b]$ into n pieces of equal width $\frac{b-a}{n}$, traditionally abbreviated as Δx.

Let x_i^* be some x value in the ith subinterval.

Then $\sum_{i=1}^{n} f(x_i^*)\Delta x$ represents a sum of areas of rectangles approximating the area under the curve.

The "definite integral"

If the limit of the Riemann sums exists as $n \to \infty$, the function f is called *integrable* on the interval $[a, b]$, and the limit is declared to be the area under the curve.

Notation:

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\, dx$$

If the graph is sometimes above the x-axis and sometimes below, then this construction computes *signed* area: the area below the axis is treated as negative.

Exercises for this week (not to hand in)

- Section 5.1: Exercises 7, 13, 17, 21, 23.
- Section 5.2: Exercises 3, 7, 17, 33, 35, 37, 41, 47, 49, 55.
- Section 5.3: Exercises 3, 7, 9, 13, 17, 19, 21, 35, 43, 45, 61, 75.