Quiz solutions

February 7, 2019

- If an equation of the tangent line to the curve $y = f(x)$ at the point where $a = 2$ is $y = 4x - 5$, find $f(2)$ and $f'(2)$.

  [Exercise 21 in Section 2.7]

  Solution. If $y = 4x - 5$ and $x = 2$, then $y = 8 - 5 = 3$, so the point $(2, 3)$ lies on the tangent line. But the tangent line passes through the point $(2, f(2))$ on the graph of f, so $f(2) = 3$.

  The number $f'(2)$ means the slope of the tangent line, and the given line has slope 4, so $f'(2) = 4$.

- Find the derivative of the function $f(x) = 3x - 8$ using the definition of derivative.

  [Exercise 21 in Section 2.8]

  Solution. The derivative is the slope of the tangent line, and the given function has a graph that *is* a line, so the required value must be the slope of that line, namely, 3.

  What "using the definition of derivative" means is to write down the limit that defines the derivative, compute the limit, and verify that the limit indeed equals 3. Since no base point is specified, you can use a letter to denote a general base point. You could use the letter a as the book does, or the letter b that I used in class, or even the letter x. The details of the calculation depend on which of the two limit definitions you use.

  Method 1:

  $$\begin{aligned} f'(b) &= \lim_{x\to b} \frac{f(x) - f(b)}{x - b} = \lim_{x\to b} \frac{(3x - 8) - (3b - 8)}{x - b} = \lim_{x\to b} \frac{3x - 3b}{x - b} \\ &= \lim_{x\to b} \frac{3(x - b)}{x - b} = \lim_{x\to b} 3 = 3. \end{aligned}$$

  Method 2:

  $$f'(x) = \lim_{h\to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h\to 0} \frac{(3(x + h) - 8) - (3x - 8)}{h} = \lim_{h\to 0} \frac{3h}{h} = \lim_{h\to 0} 3 = 3.$$