Examination 1

1. Suppose $\vec{v} = \langle 0, 1\rangle$ and $\vec{w} = \langle 2, 3\rangle$. Compute the length $\left|\vec{v} - 3\vec{w}\right|$.

2. Suppose vector $\vec{v}$ has length 2, and vector $\vec{w}$ has length 14, and the angle between vectors $\vec{v}$ and $\vec{w}$ is $\pi/3$ radians (equivalently, 60 degrees). Determine the dot product $\vec{v} \cdot \vec{w}$.

3. Compute the vector projection of the vector $4\vec{\imath} + 3\vec{\jmath}$ onto the vector $\vec{\imath} + 2\vec{\jmath}$.

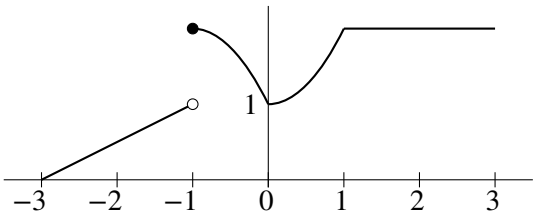
4. Compute the following limit: $\lim\limits_{x\to 2} \dfrac{x^2 - 4}{x^2 + 2x - 2}$.

5. (a) State the precise definition of: "$\lim\limits_{x\to 2} f(x) = 3$." Begin your statement as follows: "For every positive number ε, ...".

   (b) Use this definition to prove that $\lim\limits_{x\to 2}(5 - x) = 3$.

6. Sketch the graph of a function satisfying all of the following properties: $\lim\limits_{x\to -\infty} f(x) = 1$, $\lim\limits_{x\to -1} f(x) = \infty$, $f(0) = 0$, $\lim\limits_{x\to 1^-} f(x) = -\infty$, $\lim\limits_{x\to 1^+} f(x) = 2$, the function f is continuous from the right at 1, and $\lim\limits_{x\to\infty} f(x) = 0$.

7. Consider the graph of the function f shown below.

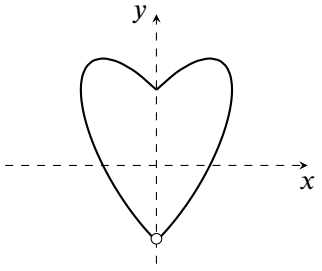


   (a) At which numbers between -3 and 3 is the function *not* differentiable?

   (b) Sketch the graph of f' (that is, the derivative of f).

8. **Optional extra-credit problem for Valentine's Day**

   The graph below is represented by parametric equations: $x = \dfrac{2t}{1 + t^2}$ and $y = \dfrac{1 + 2|t| - t^2}{1 + t^2}$ (the parameter t being an unrestricted real number).



   Find the coordinates of the points on the graph at which the tangent line is vertical.