Examination 2

Instructions. Your solution to each problem should include at least one complete sentence. If you make a computation, please state your strategy. (For example: "Now I calculate the first derivative by applying the quotient rule.")

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x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Table of values for Problems 1 and 2

1. Suppose $h(x) = f(g(x))$. Use the table above to determine $h'(2)$.
2. Use the table of values above to explain why there must be some real number x for which $f''(x)$ is equal to 0. Assume that the second derivative $f''(x)$ exists and is continuous for every real number x.
3. Find the slope of the curve $x^{42} + xy + y^3 = 1$ at the point on the curve where $x = 1$.
4. The parametric equations $x = t^3 - 2t$ and $y = 10t^3 + 6t^2$ determine a curve. Find an equation for the line tangent to the curve at the point on the curve where $t = 1$.
5. Determine the maximum value of the polynomial $x^4 - 4x^3 + 4x^2 + 41$ on the interval where $0 \le x \le 2$.
6. Sketch the graph of a function f satisfying all of the following properties: $f'(x) = 1$ when $x < -1$; $f'(x) < 0$ when $-1 < x < 0$; $f'(0) = 0$; $f'(x) > 0$ when $0 < x < 2$; $\lim_{x\to 2^-} f'(x) = \infty$; $\lim_{x\to 2^+} f'(x) = -\infty$; $f'(x) < 0$ when $2 < x < 4$; $f'(4) = 0$; and $f'(x) < 0$ when $x > 4$.
7. When x is a small positive number, is e^{-42x} larger than $1 - 42x$ or smaller than $1 - 42x$? Explain how you know.
8. *Optional extra-credit problem for March Madness.*
   Suppose the volume of a sphere is increasing at a rate of $(48/7)$ cm^3/sec. How fast is the circumference of the sphere changing when the radius is 12 cm?

   Remark. This problem is motivated by the current NCAA basketball tournament, in which the TAMU women's team has advanced to the third round. The size of a basketball is commonly stated in terms of the circumference, which equals 2π times the radius. The volume of a sphere equals $\frac{4}{3}\pi$ times the cube of the radius. A men's basketball has a radius of about 12 cm, and a women's basketball has a radius about half a centimeter smaller.