Math 171 Calculus Spring 2019

Final Examination

Instructions. Your solution to each problem should include at least one complete sentence. When making a computation, please state your strategy. (For example: “Now I calculate the first derivative by applying the quotient rule.”)

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1. Which of the following three numbers is smallest? Explain how you know.

   (a) $|\langle 1, -2\rangle|$ (b) $3\vec{\imath}\cdot(4\vec{\imath}-5\vec{\jmath})$ (c) $\lim_{x\to 0}\dfrac{e^{6x}-1}{7x}$

2. Suppose the position vector $\vec{r}(t)$ of a curve is $\ln(t)\vec{\imath}+\sin(\pi t)\vec{\jmath}$ when $t > 0$. Find an equation of the line tangent to the curve at the point where $t = 1$.

3. Consider the slope $\dfrac{dy}{dx}$ at the point on the graph where $x = 0$. For which of the following equations is that slope largest? Explain how you know.

   (a) $y = \dfrac{1+x}{1-x}$ (b) $y = x\tan(x)$ (c) $x^2 + xy + y^3 = 1$

4. Sketch the graph of a function having all of the following properties: the first derivative is positive when $x < 0$; the second derivative is negative when $x < 0$; the function has a discontinuity when $x = 0$; there is a local minimum when $x = 1$; there is an inflection point when $x = 2$; and there is a horizontal asymptote when $x \to +\infty$.

5. Which of the following integrals is largest? Explain how you know.

   (a) $\displaystyle\int_0^1 x^2\,dx$ (b) $\displaystyle\int_0^1 \sqrt{x}\,dx$ (c) $\displaystyle\int_0^1 \frac{x}{(1+x^2)^2}\,dx$

6. Is there a positive value of x for which $x + \cos(x) = 0$? Explain why or why not.

7. State *two* of the following three theorems.
   (a) the squeeze theorem for limits
   (b) the intermediate-value theorem
   (c) the mean-value theorem

8. *Optional extra-credit problem.* Suppose $f(x) = xe^{-x}$, and let $A(t)$ denote the area under the graph of f between $x = 0$ and $x = t$, as indicated in the diagram. Determine $\dfrac{dA}{dt}$, the rate of change of the area, at the value of t for which $f(t)$ is maximal.