# Quiz solutions

January 17, 2019

1. Suppose $\vec{v}$ is a vector. For each of the following quantities, say if it is a vector, a scalar, or a meaningless expression.

   (a) $5\vec{v}$ (b) $5 + \vec{v}$ (c) $5 + |\vec{v}|$ (d) $|5\vec{v} \cdot \vec{v}|$

   **Solution.** (a) vector (b) meaningless (c) scalar (d) scalar

2. Suppose $\vec{v} + \vec{w} = \langle 3, -3 \rangle$ and $\vec{v} - \vec{w} = \langle -1, 3 \rangle$.

   Find $\vec{v} + 2\vec{w}$.

   **Solution.** Adding the two equations shows that $2\vec{v} = \langle 2, 0 \rangle$, so $\vec{v} = \langle 1, 0 \rangle$. Substituting this value into the first equation and solving for $\vec{w}$ shows that $\vec{w} = \langle 2, -3 \rangle$. Therefore $\vec{v} + 2\vec{w} = \langle 1, 0 \rangle + \langle 4, -6 \rangle = \langle 5, -6 \rangle$.

3. Suppose $\vec{v} = 3\vec{\imath} + 4\vec{\jmath}$, and $\vec{v} \cdot \vec{w} = 0$, and $|\vec{w}| = 1$. There are two solutions for $\vec{w}$. Find both of them.

   **Solution.** If $\vec{w} = w_1 \vec{\imath} + w_2 \vec{\jmath}$, then $\vec{v} \cdot \vec{w} = 3w_1 + 4w_2$. So $\vec{v} \cdot \vec{w} = 0$ precisely when $3w_1 = -4w_2$. There are infinitely many choices of $w_1$ and $w_2$ that make this equation true: $w_1 = 4t$ and $w_2 = -3t$, where $t$ is any number.

   At this point what is known is that $\vec{w}$ has the form $\langle 4t, -3t \rangle$. But there is the additional constraint that $|\vec{w}| = 1$. This additional property holds when $t = \pm 1/5$. So the two solutions are $\vec{w} = \langle 4/5, -3/5 \rangle$ and $\vec{w} = \langle -4/5, 3/5 \rangle$.