1. For each of the following functions, say whether the function is (i) injective, (ii) surjective, (iii) invertible.

   (a) $f\colon \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ for every real number x.

   (b) $g\colon \mathbb{Z}^+ \to \mathbb{Z}^+$ defined by $g(n) = n^3$ for every integer n.

   (c) $h\colon \mathbb{P}(A) \to \mathbb{P}(A)$ (where A is some non-empty set, and $\mathbb{P}(A)$ is its power set) defined by $h(X) = A - X$ for every subset X of A.

2. For each of the following relations, say whether the relation is (i) reflexive, (ii) symmetric, (iii) transitive.

   (a) The relation $\leq$ on the real numbers $\mathbb{R}$.

   (b) The relation R defined on the positive integers $\mathbb{Z}^+$ by nRm if n and m have a common prime factor (that is, $\gcd(n, m) > 1$).

   (c) The relation S defined on the power set $\mathbb{P}(A)$ of a non-empty set A by XSY if $X \cup Y = A$.

3. For each of the following binary operations, say whether the operation (i) is commutative, (ii) is associative, (iii) has an identity element.

   (a) Addition on the set $\mathbb{Z}^+$ of positive integers.

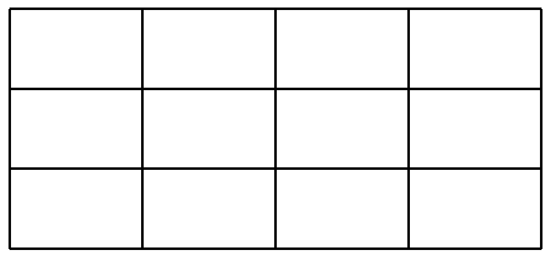
   (b) Matrix multiplication on the set $M_2(\mathbb{R})$ of 2×2 matrices with real entries.

   (c) Function composition on the set of permutations of $\{1, 2, 3\}$. (Recall that permutations of a set A are bijective functions from A to A.)

4. Working in $\mathbb{Z}_5$, the set of equivalence classes of integers modulo 5 equipped with its usual binary operations of addition (+) and multiplication ($\times$), match each entry in the first column with an equal entry in the second column.

$[4] + [19]$	$[0]$
$[13] \times [7]$	$[1]$
$[25]$	$[-1]$
$[4]^{-1}$	$[3]$

5. Consider a rectangular chocolate bar that has horizontal and vertical lines scored across the surface to mark off n small rectangles. (The figure shows the case when $n = 12$.)



You wish to break the chocolate bar into the n individual rectangles by breaking along the lines. Use the method of strong induction to show that exactly $(n - 1)$ breaks have to be made.

Extra credit

Suppose that A is a set with n elements, where n is an integer and $n \geq 3$. The following three quantities all have the same value:

(a) the number of partitions of A into three non-empty, non-overlapping subsets;

(b) the number of equivalence relations on A that have exactly three equivalence classes;

(c) one-sixth the number of surjective functions with domain A and codomain $\{1, 2, 3\}$.

Find a formula (in terms of n) for the common value of these three quantities.