Reminder

Exam 2 takes place next Tuesday, March 28.

The exam covers Chapters 3 and 4
(functions, binary operations, and relations).

Vocabulary today

- relation
- reflexive
- symmetric
- antisymmetric
- transitive
- equivalence relation
- equivalence class
- partial ordering
- linear ordering

Functions and relations

If $f : A \to A$ is a function, then each element a is paired with exactly one element $f(a)$.
A *relation* on a set A allows each element a to be paired with multiple elements (or with none at all).

Example (the birth-year relation)

$A =$ the set of Aggies; two elements of the set are related if they have the same birth year.

Formal definition of a relation

A relation on a set A is a subset of the Cartesian product $A \times A$.

Notation: aRb means a is related to b. In other words, the ordered pair (a, b) belongs to the relation.

Four properties that a relation might or might not have

- reflexive: aRa for every element a of the set A
  Example: birth year
- symmetric: aRb if and only if bRa
  Example: birth year
- transitive: if aRb and bRc then aRc
  Example: birth year
- antisymmetric: if aRb and bRa then $a = b$
  Example: $A = \mathbf{Z}$, relation $\leq$

Exercise

Are the following relations on **Z** reflexive? symmetric? transitive? antisymmetric?

1. aRb if $a \geq b$
2. aRb if $a < b$
3. clock arithmetic: aRb if a and b differ by a multiple of 12
4. parity: aRb if both a and b are even, or both a and b are odd

Answers:

1. reflexive, transitive, antisymmetric
2. transitive and antisymmetric (for a subtle reason)
3. reflexive, symmetric, and transitive
4. reflexive, symmetric, and transitive

Equivalence relations

A relation that is simultaneously reflexive, symmetric, and transitive is called an *equivalence relation*.

Example

The last two examples: clock arithmetic and parity.

Equivalence classes

If R is an equivalence relation, then one usually writes $a \sim b$ instead of aRb.
And one says "a is equivalent to b" instead of "a is related to b."

The set of all elements equivalent to a is the *equivalence class* of a, usually written $[a]$.

Example: for clock arithmetic, $[7] = \{\ldots, -17, -5, 7, 19, 31, \ldots\}$.
Notice $[7] = [-5]$.

Partial ordering

A relation that is simultaneously reflexive, antisymmetric, and transitive is called a *partial ordering*.

Example

Suppose $A = \{1, 2, 3, 4\}$, and define a relation on $\mathbf{P}(A)$ (the power set of A) by XRY if $X \subseteq Y$.
If $X = \{1, 2\}$ and $Y = \{3, 4\}$, then neither set is related to the other. Hence the term *partial* ordering.

Linear ordering

A partial ordering is called a *linear ordering* if every two elements can be compared: either aRb or bRa.

Example

$A = \mathbf{R}$ with the order relation $\leq$