All positive integers are interesting. ☺

Proof.

1 is interesting (the smallest positive integer)

2 is interesting (the smallest prime number)

3 is interesting (the smallest odd prime number)

. . .

If there were some uninteresting positive integers, then there would be a smallest one (by the well-ordering principle).

Being the *smallest* dull number, that number would be interesting!

Therefore there cannot be any uninteresting numbers after all. □

Greatest common divisor

Example

Find the greatest common divisor (gcd) of the two numbers 1333 and 1247.

Solution

Observe that $1333 = 1247 + 86$.
This equation implies that every factor of 1247 and 86 is also a factor of 1333, and every factor of 1333 and 1247 is also a factor of 86. Therefore $\gcd(1333, 1247) = \gcd(1247, 86)$.
Keep going with the same idea:

$$1247 = 14 \times 86 + 43,$$

so $\gcd(1247, 86) = \gcd(86, 43) = \gcd(2 \times 43, 43) = 43$.

This method is called the *Euclidean algorithm*.

More on the Euclidean algorithm

Example (continuation)

Find integers x and y (possibly negative) such that $1333x + 1247y = 43$.

Solution

Read the previous calculation backward:

$$\begin{aligned} 43 &= 1247 - 14 \times 86 \\ &= 1247 - 14 \times (1333 - 1247) \\ &= 1333 \times (-14) + 1247 \times 15. \end{aligned}$$

So $x = -14$ and $y = 15$.

Remark

In general, $\gcd(a, b)$ is the smallest positive linear combination of the integers a and b. [Theorem 5.3.5]

A warning about notation

Often $\gcd(a, b)$ is abbreviated simply as (a, b).

So the notation (a, b) can mean three completely different things, depending on context:

1. the greatest common divisor of integers a and b
2. an ordered pair (a, b), an element of the Cartesian product $A \times B$ of two sets
3. an interval (a, b) in the real numbers, $\{ x \in \mathbb{R} \mid a < x < b \}$