Sample outline for a paper on calculus

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1. Introduction.

   Calculus is a key tool both within mathematics and in engineering applications.

   (a) History.

      i. Already in the ancient Greek world, mathematicians such as Archimedes developed tools for computing areas and volumes, foreshadowing the theory of integration.

      ii. In the second half of the 17th century, Isaac Newton and Gottfried Wilhelm Leibniz independently developed the ideas that define calculus as it is understood today.

      iii. Mathematicians of the 19th century, such as A. L. Cauchy, B. Riemann, and K. Weierstrass, created a rigorous mathematical foundation for calculus.

   (b) Applications.
      Calculus has extensive applications in physics, chemistry, biology, medicine, and economics.

2. Differential calculus.

   (a) The derivative can be understood in multiple ways: as a slope, as a rate of change, and as a best linear approximation.

   (b) Some tools for computing derivatives are the product rule, the quotient rule, and the chain rule.

   (c) Derivatives can be used to solve optimization problems, to understand concavity, to compute limits, and to model marginal cost.

(d) Many fundamental problems in physics are modeled by differential equations.

3. Integral calculus.

   (a) Integration can be understood as the inverse process to differentiation or as a limiting process for computing a quantity through successive approximations by simpler quantities.

   (b) Some tools for computing integrals are the method of substitution and the method of integration by parts.

   (c) Integrals can be used to determine areas, volumes, lengths of curves, area of surfaces, average values, centers of mass, and work done by a force.

   (d) The fundamental theorem of calculus formalizes the concept that integration is the inverse process to differentiation.

4. Conclusion.

   Calculus is a branch of mathematics that is essential to such varied aspects of modern life as predicting the weather, pricing stock-market options, and designing airplanes.

5. References.

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   - Judith V. Grabiner, *The origins of Cauchy's rigorous calculus*, Cambridge: MIT Press, 1981.
   - Isaac Newton, *Philosophiae naturalis principia mathematica*, London: Streater, 1687.
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