# Linear Algebra

Write your **name**: ________________________________ (2 points).

In **problems 1–5**, circle the correct answer. (5 points per problem)

1. Every linearly independent set of three vectors in $R^3$ is a basis for $R^3$.
   `True` `False`

2. For $n \times n$ matrices $A$ and $B$, the determinant of the product $AB$ always equals the determinant of $BA$. `True` `False`

3. Every orthogonal $3 \times 3$ matrix has rank 3. `True` `False`

4. In an inner product space, $\|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$ for all vectors $\mathbf{u}$ and $\mathbf{v}$. `True` `False`

5. If $L$ is a linear transformation mapping $R^3$ into $R^2$, then there is a $2 \times 3$ matrix $A$ such that $L(\mathbf{x}) = A\mathbf{x}$ for every vector $\mathbf{x}$ in $R^3$.
   `True` `False`

In **problems 6–9**, fill in the blanks. (7 points per problem)

6. If $A$ is a square matrix, $\lambda$ is a scalar, and $\mathbf{x}$ is a nonzero vector such that $A\mathbf{x} = \lambda\mathbf{x}$, then $\mathbf{x}$ is called ________________________ .

7. $\det \begin{pmatrix} 3 & 0 & 4 \\ 5 & 0 & 2 \\ 8 & \square & 6 \end{pmatrix} = 28.$

8. Vectors $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$ form an orthonormal basis for $R^3$.

9. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$, then $\dim N(A)$, the nullity of $A$, equals ____ .

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In **problems 10–12**, show your work and explain your method.
(15 points per problem)

10. Suppose $A = \begin{pmatrix} 1 & -6 \\ 3 & 12 \end{pmatrix}$. Find a lower-triangular matrix $L$ and an upper-triangular matrix $U$ such that $A = LU$.

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11. Find the least-squares solution to the following inconsistent system.

$$\begin{aligned} 2x_1 + x_2 &= 5 \\ x_1 - x_2 &= 0 \\ x_1 - x_2 &= 2 \end{aligned}$$

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12. Suppose $A = \begin{pmatrix} -2 & -2 \\ 15 & 9 \end{pmatrix}$. Find a diagonal matrix $D$ and an invertible matrix $S$ such that $S^{-1}AS = D$.