Math 308.513 Exam 1 18 February 1994

1. (25 points) Find the general solution of the first-order linear differential equation $y' - 2y = x^2 e^{2x}$.
2. (25 points) Find the solution of the second-order, linear, nonhomogeneous differential equation $y'' + y' - 2y = 2x$ that satisfies the initial conditions $y(0) = 0$ and $y'(0) = 1$.
3. (25 points) Consider for $x > 0$ the second-order, linear, nonhomogeneous differential equation $x^2y'' + xy' - y = \dfrac{4}{x}$. Use the method of variation of parameters to find a particular solution of this equation. You may take as given that the general solution of the associated homogeneous equation is $y = c_1 x + \dfrac{c_2}{x}$.
4. (12 points) Consider the function $f(x) = x^2$.
   (a) Write down one specific example of a function $g(x)$ such that f and g are linearly independent functions. Explain how you know that your example works.
   (b) Now write down one specific example of a function $h(x)$ such that f and h are linearly dependent functions. Explain how you know that your example works.
5. (13 points) Your answer to this question should be written in sentences, not in formulas. Consider the first-order nonlinear differential equation $\dfrac{dy}{dx} = (y-1)(y-2)(y-3)$. **Do not solve the equation.**
   (a) Discuss the stability of the equilibrium solutions of this equation.
   (b) Suppose y represents the cockroach population of Bryan/College Station (in millions), and x represents time. What can you deduce about the long-term behavior of the cockroach population?
   (c) Describe in words how you would go about solving this differential equation (but do not actually solve it).