Math 308.513 **Exam 2** **25 March 1994**

1. (20 points) Find the eigenvalues of the matrix $\begin{pmatrix} 3 & -4 \\ 1 & 8 \end{pmatrix}$.
2. (20 points) Consider a linear system of first-order differential equations written in matrix form as $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$, where $\mathbf{A}$ is a constant matrix. Suppose that the eigenvalues of $\mathbf{A}$ are -5 and 6, and the corresponding eigenvectors are $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 9 \end{pmatrix}$.
   (a) Find the general solution $\vec{x}(t)$ of the system of differential equations.
   (b) Find the solution $\vec{x}(t)$ satisfying the initial condition $\vec{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
3. (20 points) Find the general solution of the nonhomogeneous system of first-order differential equations $\frac{d\vec{x}}{dt} = \begin{pmatrix} 7 & -24 \\ 2 & -7 \end{pmatrix}\vec{x} + \begin{pmatrix} 2e^t \\ 0 \end{pmatrix}$. You may take as given information that $\Psi(t) = \begin{pmatrix} 4e^t & 3e^{-t} \\ e^t & e^{-t} \end{pmatrix}$ is a fundamental matrix for the associated homogeneous problem, and that $\Psi(t)^{-1} = \begin{pmatrix} e^{-t} & -3e^{-t} \\ -e^t & 4e^t \end{pmatrix}$.
4. (20 points) Give a specific example of a non-diagonal matrix $\mathbf{A}$ such that $\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ with corresponding eigenvalue $\lambda = 3$. Explain how you know that your example works.
5. (20 points) Consider the differential equation $\frac{d\vec{x}}{dt} = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}\vec{x}$. You may take as given that one eigenvalue is i, with eigenvector $\begin{pmatrix} 5 \\ 2-i \end{pmatrix}$, and the other eigenvalue is $-i$, with eigenvector $\begin{pmatrix} 5 \\ 2+i \end{pmatrix}$.
   (a) Find the general solution of the differential equation in *real form* (not complex form).
   (b) If the first component of the vector $\vec{x}(t)$ represents the amount by which your bank balance exceeds \$300, and the second component represents the amount by which your weight exceeds 140 pounds, are you happy or unhappy as $t \to \infty$? Explain why.