Math 308.513 Exam 3 18 April 1994

1. (25 points) (a) Determine the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n}{n} x^n$.

   (b) Rewrite the series by shifting the summation index so that the general term involves x^{n-1} instead of x^n.

2. (25 points) Find the general solution for $x > 0$ of the Euler equidimensional equation

$$x^2 y'' - 5xy' + 9y = 0.$$

3. (25 points) Find the values of r for which the equation $2x(x+2)y'' + y' - xy = 0$ has a solution of the form $x^r \sum_{n=0}^{\infty} a_n x^n$ (with the leading coefficient $a_0 \neq 0$). **Do not solve for the coefficients a_n.**

4. (25 points) The equation $y'' - 2xy' + 8y = 0$ has a solution that is a polynomial. (It has a second solution that is not a polynomial.) Find the polynomial solution. This solution, properly normalized, is known as the Hermite polynomial $H_4(x)$.