Math 308.513 Final Exam 10 May 1994

1. (20 points) Solve the first-order linear differential equation $x\dfrac{dy}{dx} + y = e^x$ with the initial condition $y(1) = 1$.

2. (20 points) Find the general solution of the linear system of differential equations

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & 6 & 2 \end{pmatrix} \vec{x}.$$

3. (20 points) Find the first three non-zero terms in the series expansion $\displaystyle\sum_{n=0}^{\infty} a_n x^{n+r}$ of each of two linearly independent solutions of the differential equation $2x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + xy = 0$.

4. (20 points) Use Laplace transforms to solve the initial value problem $y'' + y = f(t)$, $y(0) = 0$, $y'(0) = 1$, where $f(t) = \begin{cases} 1, & 0 \le t < \pi/2 \\ 0, & \pi/2 \le t < \infty \end{cases}$.

5. (20 points) Find the general solution of the inhomogeneous second-order linear differential equation $x^2\dfrac{d^2y}{dx^2} - 2y = 4x$.