Math 311-102

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Reminder

The comprehensive final exam is 1:00–3:00PM, Tuesday, July 5, in this room.

Please bring paper (or a bluebook) to the exam.

The exam covers everything on the syllabus.

There are 10 questions on the exam.

Stokes's theorem

If the closed curve C is the boundary (border) of a surface S, then $\int_C \vec{F} \cdot d\vec{x} = \iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$.

The orientations of C and S should be compatible: as you traverse C, the positive side of S should be on your left.

Example. If S is the surface defined by $z = 1 - x^2 - y^2$ for $z > 0$, and $\vec{F}(x, y, z) = -y\vec{i} + x\vec{j} + xyz\vec{k}$, find $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$, where the surface S is oriented with its upward pointing normal.

Solution. You *could* work out the curl of $\vec{F}$ and compute the surface integral as it stands. Easier is to rewrite the problem by Stokes's theorem as $\int_C \vec{F} \cdot d\vec{x}$, where C is the circle $x^2 + y^2 = 1$ in the xy-plane. That integral equals $\int_C (-y\,dx + x\,dy)$, which by Green's theorem equals twice the area of the circle, or 2π.

Remark. The theorem shows that $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ depends only on the boundary curve C, not on S itself.