# Topics in Applied Mathematics I

In solving the following problems, you may use any method other than "My calculator says so." To obtain full credit, show your work.

1. Suppose $A = \begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & 8 \\ 0 & 2 \end{pmatrix}$. Compute $A + 2B$.

   Addition and scalar multiplication act componentwise, so $A + 2B = \begin{pmatrix} 1+2\times 9 & 0+2\times 8 \\ 4+2\times 0 & 5+2\times 2 \end{pmatrix} = \begin{pmatrix} 19 & 16 \\ 4 & 9 \end{pmatrix}$.

2. If $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 \\ 0 & 5 \\ -2 & 1 \end{pmatrix}$, which of the matrix products $AB$ and $BA$ is defined? Compute that product.

   For a matrix product to make sense, the left-hand matrix must have the same number of entries in a row as the right-hand matrix has in a column. Therefore the matrix product $BA$ is the one that is defined.

   $$BA = \begin{pmatrix} 4\times 1 + 3\times 0 & 4\times(-1) + 3\times 2 \\ 0\times 1 + 5\times 0 & 0\times(-1) + 5\times 2 \\ -2\times 1 + 1\times 0 & -2\times(-1) + 1\times 2 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 0 & 10 \\ -2 & 4 \end{pmatrix}.$$

3. Either compute the inverse of the matrix $\begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix}$ or show that the inverse does not exist.

   (This is exercise 2 on page 86 of the textbook.)

   Since the determinant of the matrix is $3\times 4 - 6\times 2 = 12 - 12 = 0$, the matrix is not invertible.

4. Find the determinant of the matrix $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 3 & 1 & 4 \\ 1 & 1 & 4 & 0 \\ -1 & -1 & 2 & 3 \end{pmatrix}$.

   (This is exercise 2 on page 98 of the textbook.)

   One could directly expand the determinant, but easier is to use row operations to simplify the calculation. For example, subtract the first row from the third row, and add the first row to the fourth row to get $\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 3 & 0 \\ 0 & -1 & 3 & 3 \end{vmatrix}$. Expand on the first column to get $\begin{vmatrix} 3 & 1 & 4 \\ 1 & 3 & 0 \\ -1 & 3 & 3 \end{vmatrix}$. Add the second row to the third row, and subtract 3 times the second row from the first row to get $\begin{vmatrix} 0 & -8 & 4 \\ 1 & 3 & 0 \\ 0 & 6 & 3 \end{vmatrix}$. Expand on the first column to get $-\begin{vmatrix} -8 & 4 \\ 6 & 3 \end{vmatrix}$. Factor out $-4$ from the first row and 3 from the second row to get $12\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = 12(2-(-2)) = 48$.

5. Find the inverse of the matrix $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$.

   (This is exercise 20 on page 99 of the textbook.)

   Using the row-reduction algorithm for finding the inverse matrix, divide each row by its leading entry:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix}.$$

   Subtract the third row from the first row:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Bigg| \begin{pmatrix} 1 & 0 & -1/3 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix}.$$

   Therefore the inverse matrix equals $\begin{pmatrix} 1 & 0 & -1/3 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix}$.