Final Exam Math 452 9 May 1994

1. Solve the following (nonstandard) problem about temperature in a rod:

$$\frac{\partial^2 u}{\partial x^2} = 2u + \frac{\partial u}{\partial t}, \qquad 0 < x < 1, \quad 0 < t,$$

with the boundary conditions

$$\frac{\partial u}{\partial x}(0,t) = 0, \qquad u(1,t) = 0, \qquad 0 < t,$$

and the initial condition $u(x,0) = 5$.

2. The discontinuous odd function

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ -1, & -1 < x < 0 \end{cases}$$

on the interval $(-1,1)$ has a Fourier series

$$b_1 \sin(\pi x) + b_3 \sin(3\pi x) + b_5 \sin(5\pi x) + \ldots$$

as well as a generalized Fourier series

$$c_1 P_1(x) + c_3 P_3(x) + c_5 P_5(x) + \ldots$$

in terms of Legendre polynomials. Suppose both series are truncated after the first two (nonzero) terms. Which one gives a better mean-square approximation to f? In other words, determine if

$$\int_{-1}^{1} \big(f(x) - b_1 \sin(\pi x) - b_3 \sin(3\pi x)\big)^2 \, dx$$

is bigger or smaller than

$$\int_{-1}^{1} \big(f(x) - c_1 P_1(x) - c_3 P_3(x)\big)^2 \, dx.$$