The superposition principle

The superposition principle says that a linear combination of solutions of a differential equation is again a solution—under certain conditions. The goal of this exercise is to understand when the superposition principle applies to a differential equation. You will also review the solution of some basic differential equations from Math 308.

1. Find the general solution of the following ordinary differential equations. In each case, the letter k denotes a real constant.

   (a) $\dfrac{dy}{dx} + kx = 0$

   (b) $\dfrac{dy}{dx} + ky = 0$

   (c) $\dfrac{dy}{dx} - y + kx = 0$

   (d) $\dfrac{dy}{dx} + ky^2 = 0$

   (e) $\dfrac{d^2y}{dx^2} + k^2y = 0$

   (f) $\dfrac{d^2y}{dx^2} - k^2y = 0$

2. Solve the following differential equations with side conditions. Again, k denotes a real constant.

   (a) $\dfrac{d^2y}{dx^2} + k^2y = 0$, and $y(0) = 0$

   (b) $\dfrac{d^2y}{dx^2} - k^2y = 0$, and $y'(0) = 1$

   (c) $\dfrac{d^2y}{dx^2} + k^2y = 0$, and $y(0) = y'(1) = 2$

3. (a) In which of the above cases is it true that a linear combination[1] of solutions is again a solution?

   (b) In general, what needs to happen for the superposition principle to apply?

[1] A linear combination of two functions $f(x)$ and $g(x)$ means a function of the form $af(x) + bg(x)$, where a and b are constants.