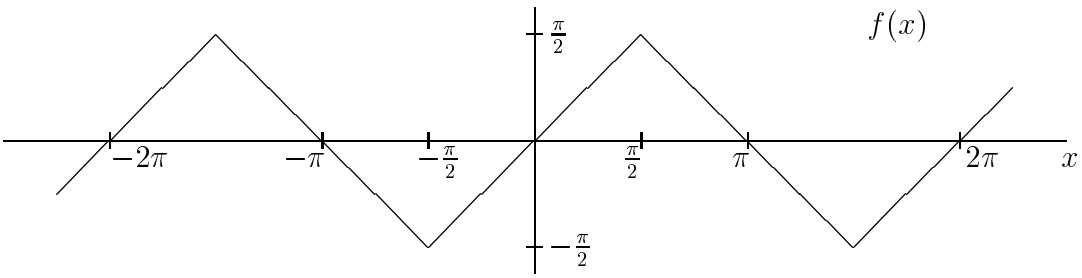
Exercise: calculating a Fourier series

In solving the heat equation on a finite interval, you found that typically the transient solution is a superposition of sines and cosines in the space variable multiplied by decaying exponential functions in the time variable.

To find the coefficients in the superposed solution, you would set time equal to zero and invoke the initial condition. That means you have to find how to express a given function as a superposition of sines and cosines. Here is an example exercise on computing a *Fourier series*.



1. Find a formula on the interval $[-\pi, \pi]$ for the periodic function $f(x)$ shown in the diagram.

2. Suppose $f(x) = \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$. Find the Fourier coefficients a_n and b_n by multiplying by an appropriate sine or cosine, integrating, and using the orthogonality relations.

3. Why is there no constant term in this Fourier series?